

PHILOSOPHICAL TRANSACTIONS.

- I. *On the Laws of the Tides on the Coasts of Ireland, as inferred from an extensive series of observations made in connection with the Ordnance Survey of Ireland.*
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Received November 30, 1844,—Read December 12, 1844.

IN the spring of 1842 I was informed by Colonel COLBY, R.E., Director of the Trigonometrical Survey, that in the operations of the Survey of Ireland it had become necessary to adopt a line of reference for the elevations ascertained in the running of various lines of level through the country; and that it was his intention to institute a series of observations of the height of the water in different states of the tide, in order to refer the levels to the mean height of the sea, or to its height at some definite phase of the tide. Colonel COLBY stated also that he was desirous that the observations should be made subservient to improvements in the theory of the tides, and requested my assistance in sketching a plan of observation which would be most likely to contribute to that end.

In reply, I made the following suggestions:—That great care should be taken in the accurate determination of time at every station, and that for this purpose the non-commissioned officer of the Royal Sappers and Miners who had the care of the observations at each station, should be entrusted with a pocket chronometer, and that an officer should, at least twice during the series of observations, visit every station, carrying, for comparison, an itinerant chronometer whose error on Greenwich time was accurately known from astronomical observations. That stations should be chosen on the eastern as well as on the western coast, in order to determine the difference of level, if any, between an open sea and a partially inclosed sea. That on the north-eastern coast, stations should be selected at smaller intermediate distances than at other parts of the coast, with the purpose of removing, if possible, the doubt which appears to exist as to the progress of the semidiurnal tide-wave through the North Channel. That, where practicable, several stations should be selected on each of the large rivers or estuaries, in order to ascertain the nature of the modification

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which the tide-wave undergoes in passing up a contracted channel of comparatively small depth. That the series of observations should be so arranged, that, at every station, one complete tide (from high water to high water, or from low water to low water) should be completely observed on every day, its observations being made at small equidistant intervals. That supplementary observations, applying only to the neighbourhood of the low water or high water omitted in the observations of the complete tide, should also be made, for the development of the principal facts of diurnal tide. Finally, that the zeros of the tide-gauges should be connected with the principal lines of level, so that every observation should be referred to the same hydrostatic level.

These suggestions were adopted in their utmost extent by Colonel COLBY. The collection of observations was placed in my hands in the winter of 1842. The whole number of observations exceeds two hundred thousand; the circumstances of place, simultaneity, extent of plan, and uniformity of plan, appear to give them extraordinary value; and extent of time alone appears wanting to render them the most important series of tide-observations that has ever been made.

Having under my immediate direction a large number of computers, employed at the Royal Observatory under the authority of the Lords Commissioners of the Treasury for the reduction of the Greenwich Lunar Observations, I requested the sanction of their Lordships for the employment of a part of this force on the reduction of these tide observations. With this request their Lordships were pleased to comply; and the investigations and results which I have now the honour to lay before the Royal Society are the fruit of this liberality.

The following is the order in which I propose to arrange the parts of this memoir:—

Section I.—Account of the stations, levellings, times, and methods of observation.

Section II.—Methods of extracting from the observations the times of high and low water; of supplying deficient times and heights; and of correcting the times first determined.

Section III.—Theory of diurnal tide as related to observations only; and deduction of the principal results for diurnal tide given immediately by these observations.

Section IV.—Theory of diurnal tide as referred to the actions of the sun and moon.

Section V.—Discussion of the height of apparent mean water, as deduced from the heights of high and low water only, corrected for diurnal tide; with reference to difference of station, and to variations of the phase of the moon, and of the declination of the moon.

Section VI.—Discussion of the range of the tide, and of semimenstrual inequality in height, apparent proportion of solar and lunar effects as shown by heights, and age of tide as shown by heights; from high water and from low water.

Section VII.—Establishment of each port, and progress of semidiurnal tide round the island.

Section VIII.—Semimenstrual inequality in time; proportion of solar and lunar

effects as shown by times, and apparent age of tide as shown by times ; from high water and from low water.

Section IX.—Formation of the time of diurnal high water ; progress of the diurnal tide-wave round the island ; comparison of its progress and range with those of the semidiurnal tide.

Section X.—Method of expressing the height of the water, throughout every individual tide, by sines and cosines of arcs ; and expressions in this form for every tide in the whole series of observations, except those at Courtown.

Section XI.—Discussion of the height of mean water deduced from the analysis of individual tides ; with reference to difference of station, and to variations of the phase of the moon, and of the declination of the moon.

Section XII.—Discussion of range of tide, or coefficient of first arc in the analysis of individual tides ; and of semimenstrual inequality in range, apparent proportion of solar and lunar effects, and age of tide as deduced from range.

Section XIII.—Establishment of each port, as deduced from the time of maximum of the first periodical term in the analysis of individual tides.

Section XIV.—Semimenstrual inequality in time, proportion of solar and lunar effects from times, and apparent age of tide as shown by times ; deduced from the time of maximum of the first periodical term.

Section XV.—Comparison of the results as to mean height, range, semimenstrual inequality in height, age of tide obtained from height, establishment, semimenstrual inequality in time, and age of tide obtained from time, deduced from high and low waters only, in Sections V., VI., VII., VIII., with those deduced from the analysis of individual tides in Sections XI., XII., XIII., XIV.

Section XVI.—Remarks on the succeeding terms of the expressions for individual tides, as related to the magnitude of the tide, to the position on the sea-coast, to the position on the river, &c. ; comparison with the terms given by the theory of waves ; discussion of the quarto-diurnal tide.

Section XVII.—Separate discussion of the tidal observations at Courtown.

Section XVIII.—Examination into the question of tertio-diurnal tide.

Section I.—*Account of the Stations, Levellings, Times, and Methods of Observation.*

The following are the stations of observation :—

1. Kilbaha.—A small bay in the Shannon, on its north side, very near to the Loop Head ; latitude $52^{\circ} 34'$, longitude $9^{\circ} 52'$ west of Greenwich. The gauge was a graduated post erected in the sea at a short distance from the pier ; it was kept upright by large stones at its base, and by guys with large stones lashed to them.

2. Kilrush.—A small town on the north bank of the Shannon, about sixteen miles above Kilbaha ; latitude $52^{\circ} 38'$, longitude $9^{\circ} 29'$. The gauge was a graduated scale nailed to the Revenue Pier.

3. Foynes Island.—An island in the Shannon, about fifteen miles above Kilrush; latitude $52^{\circ} 37'$, longitude $9^{\circ} 9'$. The gauge was a pole in the narrow channel on the south side of the island.

4. Limerick.—The tide-gauge was on the face of Mead's Quay, the lowest (on the course of the river) of the quays at Limerick; latitude $52^{\circ} 38'$, longitude $8^{\circ} 39'$.

5. Casleh Bay.—A small bay in the north side of Galway Bay, near its entrance; latitude $53^{\circ} 14'$, longitude $9^{\circ} 34'$. The tide-gauge was a pole in the water near the Coast-Guard Station.

6. Galway.—The tide-gauge was nailed to the pier, at the entrance to the New Dock; latitude $53^{\circ} 17'$, longitude $9^{\circ} 2'$.

7. Old Head.—A station on the south side of Clew Bay; latitude $53^{\circ} 47'$, longitude $9^{\circ} 47'$. The gauge was at first nailed to a small quay; it was afterwards fixed in the water.

8. Mullaghmore.—A station on the south side of Donegal Bay; latitude $54^{\circ} 27'$, longitude $8^{\circ} 26'$. The gauge was nailed to the south pier of a small port called Classybaun Harbour.

9. Buncrana.—A station on the east side of Lough Swilly, about ten miles from its mouth; latitude $55^{\circ} 8'$, longitude $7^{\circ} 27'$. The gauge was fixed in the water, opposite to a fortress called Ned's Point Battery.

10. Port Rush.—A small harbour near the entrance of Lough Foyle; latitude $55^{\circ} 11'$, longitude $6^{\circ} 40'$. The gauge was nailed to the northern pier.

11. Carrowkeel.—A station on the western side of Lough Foyle, about twelve miles from its mouth; latitude $55^{\circ} 7'$, longitude $7^{\circ} 11'$. The gauge was nailed to a large post permanently fixed in the water.

12. Ballycastle.—A small port opposite Rathlin Island; latitude $55^{\circ} 12'$, longitude $6^{\circ} 14'$. The gauge was a pole in the water.

13. Glenarm.—A small port between Ballycastle and Belfast; latitude $54^{\circ} 56'$, longitude $5^{\circ} 56'$. The gauge was nailed to the pier.

14. Donaghadee.—Latitude $54^{\circ} 39'$, longitude $5^{\circ} 32'$; near the south side of the entrance to Belfast Lough. The gauge was nailed to the pier.

15. Ardglass.—A small harbour opposite to the Isle of Man; latitude $54^{\circ} 15'$, longitude $5^{\circ} 35'$. The gauge was nailed to the pier.

16. Clogher Head.—A headland a few miles north of Drogheda; latitude $53^{\circ} 48'$, longitude $6^{\circ} 14'$. The observations were made at a small harbour called Port Oriel, on the north side of the Head, but very near to it. The gauge was nailed to the pier.

17. Kingstown.—The harbour on the south side of Dublin Bay; latitude $53^{\circ} 18'$, longitude $6^{\circ} 9'$. The gauge was nailed to the wharf, on the landward side of the harbour.

18. Courtown.—A small harbour; latitude $52^{\circ} 38'$, longitude $6^{\circ} 14'$. The gauge was nailed to the wall of a canal which forms the opening from a small river into the sea.

19. **Dunmore East.**—A small port on the west side of the entrance of Waterford Harbour (the confluence of the rivers Barrow and Suir); latitude $52^{\circ} 9'$, longitude $6^{\circ} 59'$. The gauge was nailed to the pier.

20. **New Ross.**—A town on the river Barrow; latitude $52^{\circ} 24'$, longitude $6^{\circ} 56'$. The gauge was nailed to the bridge.

21. **Passage West.**—This is the western side of the narrow channel by which the waters of the river Lee, at Cork, principally enter into the broad harbour of Cork. Its latitude is $51^{\circ} 53'$, longitude $8^{\circ} 21'$. The tide-gauge was nailed to the face of the steam-boat wharf.

22. **Castle Townsend.**—A small harbour at very nearly the southernmost point of Ireland; latitude $51^{\circ} 30'$, longitude $9^{\circ} 20'$. The gauge was a pole in the sea.

Of these stations it may be remarked that Kilbaha, Kilrush, Foynes Island, and Limerick, are four stations in succession on the same river; that Dunmore East and New Ross are on the same river; and that Port Rush and Carrowkeel are nearly in the same circumstances: Casleh Bay and Galway are in a relation nearly resembling this. Kilbaha, Casleh Bay, Old Head, Mullaghmore, Port Rush, Ballycastle, Glenarm, Donaghadee, Ardglass, Clogher Head, Kingstown, Courtown, Dunmore East, and Castle Townsend, are all on the open sea; Kilrush, Galway, Buncrana, Carrowkeel, and Passage West, are somewhat removed from it.

The zero of the tide-gauge at each place was referred as early as possible in the course of the observations to a permanent mark (usually a copper bolt driven into the face of a rock), and these marks were all connected by a system of levellings, and thus the zeros of the tide-gauges were all referred to the same mark, namely, a copper bolt fixed in the upper surface of the heelstone or hinge-stone of the gate of Buckingham Lock, Dublin. To give the means of recovering this important zero, in the event of its loss, the following references were made to several public buildings in or near Dublin:—

Building.	Position of the mark on the building.	Elevation of the mark above the bolt at Buckingham Lock.
		feet.
Four Courts	Pavement or floor of portico at principal entrance	+ 0·928
General Post-Office	Pavement or floor of portico	+ 3·145
Bank of Ireland	Pavement or floor of colonnade at principal entrance	+ 6·113
Custom House	Pavement or floor of portico at principal entrance	+ 2·195
Carlisle Bridge	Copper bolt driven horizontally into the stone-work of the battlement, 3·900 feet below the top of the battlement, and 2·840 feet above the centre of the road	+ 8·705
Queen's Bridge	Copper bolt driven horizontally into the stone-work of the battlement, 4·450 feet below the top of the battlement, and 1·580 foot above the centre of the road	+ 8·019
Trinity College	An arrow cut on the stone-work at the principal entrance in College Green, 3·300 feet above the surface of the ground	+ 4·026
Poolbeg Light House	A mark on the surface of the base-course, under the south window (the mark lower than the bolt at Buckingham Lock)	— 0·020

It will be seen, in the sequel, that results are obtained relating to the relative levels of the surface of the sea at different places, which if established must be deemed highly important. In order, therefore, to show what confidence is due to the results of relative level, I think it necessary to give, for every instance in which the means of doing so exist, the amount of discrepancy between the results for the difference of level between the same stations when the space between them was traversed by different lines.

Description of marks whose difference of level is ascertained, and courses of the lines of levelling.		Approximate length of line in miles.	Difference of level		Difference of backward and forward levelling.	Mean of backward and forward levelling.	Difference of the means by different lines.
			By forward levelling.	By backward levelling.			
1.	{ Bolt in the front of the hotel, New Ross, above the bolt at Buckingham Lock, Dublin.....	101	ft. 10-698	ft. 11-826	ft. 1-128	ft. 11-262	} 0-171
			By Monasterevan	84	11-469	11-397	
2.	{ Mark at Lord Glengall's house, Cahir, below bolt in Monasterevan Church.....	72	48-203	48-482	0-279	48-342	} 0-418
			By Borris-in-Ossery	110	48-796	47-053	
3.	{ Bolt in Penrose Quay, Cork, below mark on William Street Bridge, Waterford.....	95	3-471	3-257	0-214	3-364	} 0-630
			By Cahir	85	3-973	4-015	
4.	{ Mark on Mead's Quay, Limerick, below mark at Court House, Borris-in-Ossery.....	50	348-344	347-465	0-879	347-904	} 0-04
			By Nenagh	68	347-661	347-940	
5.	{ Mark on Tralee Bridge, below mark on Holy-cross Bridge.....	105	272-575	273-262	0-687	272-918	} 0-264
			By Limerick.....	122	272-389	272-920	
6.	{ Bolt in Loughrea Church, above the bolt at Buckingham Lock, Dublin.....	108	268-558	268-993	0-435	268-775	} 0-135
			By Ballinasloe.....	130	268-762	268-519	
7.	{ Mark on Oranmore Bridge, above mark on Mead's Quay, Limerick.....	76	7-573	7-981	0-408	7-777	} 0-336
			By Nenagh	109	8-341	7-886	
8.	{ Bolt in the Market House, Armagh, above the bolt at Buckingham Lock.....	82	146-771	147-146	0-375	146-958	} 0-147
			By Dundalk and Newry.....	118	147-061	146-561	
9.	{ Bolt in the front of Commercial Buildings, Belfast, below bolt on Sugar Island Bridge, Newry...	58	5-477	5-489	0-012	5-483	} 0-056
			By Navan, Cavan, Newtown Butler and Monaghan	62	4-970	6-109	
10.	{ Bolt in Port Rush Pier, below bolt in Commercial Buildings, Belfast	83	6-327	5-086	1-241	5-706	} 0-908
			By Glenarm and Ballycastle	62	6-334	6-893	
11.	{ Bolt in Castlebar Gaol, below mark at Kinnegad.....	140	98-589	98-341	0-248	98-465	} 0-487
			By Longford, Ballysadare and Ballina	122	98-795	99-110	
12.	{ Bolt in Commercial Buildings, Belfast, below the bolt at Buckingham Lock.....	156	0-486	1-087	0-601	0-786	} 0-091
			By Newtown Butler and Armagh.....	125	0-269	1-122	
13.	{ Mark at Londonderry Bridge, above bolt at Buckingham Lock.....	168	2-828	2-351	0-477	2-589	} 1-058
			By Newtown Butler, Monaghan and Strabane ...	243	
14.	{ Mark at Clifony, near Mullaghmore, above bolt at Buckingham Lock.....	139	93-998	93-117	0-881	93-557	} 0-140
			By lines round the east and north coasts.....	145	92-520	94-314	

On these results, the following remarks are made by Colonel COLBY.

The discordance in No. 10 is unusually great; but as it appears from an examination of the levelling that the error in discordance is of gradual accumulation and does not arise from any one mistake, the mean of the two results is adopted for Port Rush, and a part of the difference between this mean and the result on either line, proportional to the distance of any point on that line from Belfast, is adopted as a correction for the apparent elevation of such point.

In No. 13, second line, the comparison of forward and backward levelling is omitted, the principal part (namely that from Dublin to Port Rush) being contained in Nos. 10 and 12. The mean of the two results for Londonderry was adopted; upon this depend the zeros of Carrowkeel and Buncrana.

The station at Limerick being important, its relation of height to that at Dublin was ascertained by four different lines. The following Table contains the results of

these levellings. The first result, or that by the shortest line, was adopted in the reduction of the tide-observations.

Depression of the Bench-mark at Mead's Quay, Limerick, below the bolt at Buckingham Lock, Dublin.

Line along which the levels were carried.	Approximate length in miles.	Depression by forward levelling.	Depression by back levelling.	Mean of results forward and back.
		feet.	feet.	feet.
From Dublin by Monasterevan and Borris-in-Ossory; the difference of level from Borris-in-Ossory to Limerick being the mean of those found by Nenagh and by Holycross	127	0.629	0.651	0.640
From Dublin by Cork, Mallow, and Tralee; the difference of level from Dublin to Cork being the mean of those found by Monasterevan, Borris-in-Ossory, Cahir and Mallow, and by New Ross, Waterford and Dungarvan.	317	0.032	0.744	0.388
From Dublin by Gorey, New Ross, Waterford, Mallow and Tralee; the difference of level from Waterford to Mallow being the mean of those by Cahir and by Cork	312	0.249	-0.350	-0.050
From Dublin by Ballinasloe, Oranmore, Liscanor and Kilrush	233	0.899	0.889	0.894

In order to give the means of verifying the principal results of the tide-observations at any future time, I subjoin a statement of the positions of the bench-marks at the different tide stations, and of their difference in elevation from the bolt at Buckingham Lock, Dublin.

Tidal Station.	Description of the permanent mark.	Elevation of the mark above the bolt at Buckingham Lock.
		feet.
Kilbaha	Top of copper bolt driven vertically into one of the facing-stones of the pier . .	- 2.814
Kilrush	Top of copper bolt driven vertically into one of the facing-stones of the pier . .	- 1.974
Foynes Island.	Top of copper bolt driven vertically into the solid rock	- 2.920
Limerick.	Copper bolt driven horizontally into one of the facing-stones at Russell's Quay	- 0.850
Casleh Bay	Top of copper bolt driven vertically into the solid rock, close to the Coast-Guard Watch House.	+ 0.004
Galway	Copper bolt driven horizontally into one of the facing-stones at the entrance to the New Dock	- 3.000
Old Head	Copper bolt driven vertically into one of the facing-stones of the quay.	- 0.870
Mullaghmore	Copper bolt driven vertically into one of the facing-stones of the south pier. . . .	- 1.749
Buncrana	Copper bolt driven horizontally into the scarpwall of Ned's Point Battery	+ 18.203
Port Rush	Copper bolt driven vertically into one of the facing-stones of the quay.	- 6.900
Carrowkeel.	Copper bolt in the wall of the Police barrack	+ 24.820
Ballycastle	Copper bolt driven vertically into one of the stones of the quay.	- 2.268
Glenarm	Copper bolt driven into the solid rock on which the pier is built	- 9.388
Donaghadee	Copper bolt driven vertically into one of the facing-stones of the quay.	+ 0.321
Ardglass.	Copper bolt driven vertically into one of the facing-stones of the pier, near the steps	+ 0.838
Clogher Head.	Mark on one of the facing-stones of the pier	+ 2.567
Kingstown	Top of copper bolt driven vertically into one of the facing-stones of the pier . .	- 2.796
Courtown	Top of copper bolt driven vertically into one of the facing-stones of the entrance to the harbour	- 4.459
Dunmore East.	Top of copper bolt driven vertically into one of the facing-stones of the pier . .	- 3.711
New Ross	Mark in one of the facing-stones of the quay	- 4.178
Passage West.	Top of copper bolt driven vertically into one of the coping-stones at the edge of the pier	- 6.463
Castle Townsend	Top of iron bolt driven vertically into the rock in which the Coast-Guard signal-staff is secured	+ 10.739

The results for height in the subsequent sections of this paper are all referred to a point thirty feet below the bolt at Buckingham Lock, Dublin.

At each of the stations the course of observation was as follows:—The observer adopted, as the tide which was to be completely observed, either the interval from high water to high water, or that from low water to low water, according to the convenience of the hours. Thus, having begun, for instance, with commencing a tide at the morning high water, when the high water occurred at convenient hours both in the morning and the evening; as the tides in the succession of days fell later and later every day, the termination of the tide at last fell inconveniently late in the evening, and the observer then began his observations about six hours earlier in the morning, so as to commence with low water and to terminate with low water. After a time it became necessary, in consequence of the evening low water occurring inconveniently late, to commence again with high water; and thus there was in every few days a change in the arrangement of observations.

The observations were generally commenced about half an hour before the commencing high water or low water, and were generally continued about half an hour after the terminating high water or low water. Thus, of the four principal phases which occur in each day (two high waters and two low waters), three were effectually observed in the day series of observations. As there were at each station at least two observers, one of these persons made observations for an hour or more in the night, partly before and partly after the remaining high water or low water; and thus all the high waters and low waters were observed. This system had the advantage of giving all the phenomena of diurnal tide, and giving one semidiurnal tide completely observed in each day, with little distress to the observer. Its only disadvantage is, that the observations at different stations do not always apply to the same portions of corresponding tides; but there appears to be no method of securing this precise correspondence of observations except by incessant observations day and night, or by self-registering tide-gauges. Each observer registered the height of the water on his tide-pole at every five minutes by his watch.

The watches were for the most part chronometers or lever watches. An officer visited each station at least three times, and the greater number of the stations four times, carrying a good pocket chronometer whose error on Greenwich time was known. Two itinerant chronometers were thus employed. The error of each of the observers' watches was afterwards computed for every day of observation from these comparisons, and this error was applied to form the corrected Greenwich time of every observation, in a column purposely left in the sheets of observations.

At two stations only, Ballycastle and Glenarm, the means for registering the time proved imperfect. At the former, in consequence of the failure of the watch, the time was taken from the town-clock, and corrected for the longitude of the place; it is supposed that this time may be sometimes ten minutes in error. I much regret that the extraordinary phenomena of the tides at Ballycastle are thus developed with

less certainty than could be desired; at the same time I have no hesitation in expressing my belief, that the credit of the results hereafter to be given is not sensibly injured by this circumstance. At Glenarm, from a similar cause, it became necessary to refer to the post-office clock; but the observations do not appear to have suffered materially.

The observations began about June 22, and were discontinued about August 22.

Section II.—*Methods of extracting from the observations the times of high and low water; of supplying deficient times and heights; and of correcting the times first determined.*

The determination of the height of the water, at high or low water, from the observations, was a matter of no difficulty. In two or three instances of low water, when the water had dropped below the zero of the tide-gauge, the observations were incomplete till it again rose to the zero; in these, the observations were supplied by comparison with other low waters which had been completely observed.

The determination of the time was far more difficult. The examination of these observations has made me very distrustful of the results which have been deduced from observations of time only. The difficulty of fixing on the precise time of high or low water will appear from this statement, that sometimes twenty or twenty-four successive observations (occupying $1^h 40^m$, or 2^h) are registered with the same decimal of a foot for the height. The most perplexing case is that where the change of height, in respect to change of time, follows or may follow different laws before and after the principal phase. Thus at Limerick, after low water, the water sometimes rises as much in ten minutes as it had previously dropped in two hours; it therefore appears right here, if several successive observations about low water are registered at the same decimal of a foot, to suppose that the real low water is little before the last of those observations. At some other stations this circumstance does not happen uniformly; and then, when it does happen, it becomes difficult to say whether there is a difference of law before and after the low water (in which case the real low water ought to be taken nearer to the last observation), or whether the surface of the water at the last observations on the same division has been depressed by accident (in which case the real low water ought to be taken nearer to the first observations). I will not undertake to say that, in marking off the times of high and low water, I have followed a uniform method in these difficulties; but I have certainly followed a uniform plan for each station; and this is all that is important.

Occasionally, though rarely, observations of high and low water were interrupted by the roughness of the sea and other accidents. It was highly desirable to supply these, because (as will be seen in the next section) differences of the heights and of the times to the fourth order were to be taken, and thus the omission of one height or time would entail the loss of five results in these differences of the 4th order. The following is the process by which they were supplied. It very soon became

evident to all who inspected the collected heights at high and low water, that irregularities in the heights at any one station were sensible with no important difference of magnitude at the neighbouring stations. This will be abundantly shown in Section XI. On this assumption, a comparison of the height or time at one station with that at each of the neighbouring stations, for a few tides near to that at which the observation was deficient, would give the means of supplying the omitted height or time. But it was necessary to bear in mind that all the observations were affected by diurnal tide, and that the diurnal tide might vary sufficiently from port to port to render it unsafe to use comparisons of evening tides for the correction of morning tides, &c. The process adopted therefore was the following:—The results, both for times and for heights, were divided into four groups. One comprehended the high waters which next followed the moon's transit; these were called High Waters of the First Division. Another comprehended the low waters which next followed those high waters; these were called Low Waters of the First Division. The remaining high waters and low waters were called respectively High Waters and Low Waters of the Second Division. Each of these groups was treated separately. When a height or time of high or low water at any station was to be supplied, the observed height, &c. at that station was compared with the mean of the observed heights, &c. for at least two neighbouring stations, in at least two tides preceding and two tides following, in the same group; and the mean difference thus found was applied to the mean of the observed heights, &c. at the stations compared, on that day for which the tide was deficient. I have no doubt that the results thus supplied are sensibly as accurate as those which were actually observed.

On consideration of the difficulty of determining the times of high and low water, which has been already explained, it appeared necessary to endeavour to smooth down some of their irregularities, without at the same time endangering the conclusions as affected by difference of diurnal tides and of semidiurnal tides at the different stations. The following is the method employed:—Each of the groups already mentioned was separated into four subdivisions, determined by the proximity of stations. One included Kilbaha, Kilrush, Foynes Island, Limerick, Casleh Bay, Galway, and Old Head. The second included Mullaghmore, Buncrana, Port Rush, Carrowkeel, and Ballycastle. The third contained Glenarm, Donaghadee, Ardglass, Clogher Head, and Kingstown. The fourth contained Dunmore East, New Ross, Passage West, and Castle Townsend. [Courtown was omitted, because, as will be hereafter seen, no times of high or low water could be fixed for it.] Then each subdivision was treated separately. For each tide the mean of the times for all the different stations was taken (Buncrana, Ballycastle, and Glenarm, being excepted; as, from the small range of tide at these places, the determinations were more uncertain than at others). Then for every station (including those already named) the difference of the time from the mean of times was formed. Thus, for any one station, a difference from mean was obtained for each day. Let these differences for successive days be called

$D_1, D_2, D_3, D_4, D_5, \&c.$ Then the means of the adjacent numbers were taken,

$$\frac{D_1+D_2}{2}, \frac{D_2+D_3}{2}, \frac{D_3+D_4}{2}, \frac{D_4+D_5}{2}, \&c.;$$

and the means of the numbers in this series were taken, forming

$$\frac{D_1+2D_2+D_3}{4}, \frac{D_2+2D_3+D_4}{4}, \frac{D_3+2D_4+D_5}{4}, \&c.$$

Then the number $\frac{D_1+2D_2+D_3}{4}$ was considered to be the just difference from mean for the second day in the series: it was applied to the mean of times for that day, and gave the adopted time for high or low water for that day, at the station under consideration; and so for the succeeding days. In regard to the legitimacy of this process, it is to be observed that it does not suppress the inequalities affecting, in different degrees at different stations, the semidiurnal or diurnal tide, provided the period of such inequalities is of several days. Nor does it suppress any accidental inequality which affects the whole tide-wave coming from the Atlantic upon a large extent of coast. The only failure is, that, as

$$\frac{D_1+2D_2+D_3}{4} = D_2 + \frac{D_1-2D_2+D_3}{4} = D_2 + \frac{1}{4} (\text{2nd difference});$$

when the second difference of D is large, an error is introduced. So long as the tides at the different stations follow anything like similar laws, there is no fear that this error will be perceptible. The only place where there is any probability that it can become sensible is Ballycastle; and here it will be very far below the irregularities of observation.

Section III.—*Theory of diurnal tide as related to observations only; and deduction of the principal results for diurnal tide given immediately by these observations.*

The remarks with which I shall immediately proceed apply equally to times and to heights, and equally to high waters and to low waters; but, to avoid unnecessary repetitions, I shall speak only of heights at high water.

Suppose then that, for any station, the heights at high water, both of the First Division and of the Second Division, have been collected and intermingled in the order of times. It is evident that the diurnal tide at any one of these heights will be found approximately by taking half the excess of that height above the mean of the two heights immediately preceding and immediately following. The number thus found will, however, be in error by one-fourth of the second difference of the semidiurnal tide. This error may be eliminated, leaving only an error depending on fourth differences, by taking half the algebraical excess of that apparent diurnal tide above the mean of the diurnal tides next to it.

The process may however be put in the following algebraical form:—Suppose the successive high waters to be affected with inequalities represented by $a \cdot \cos n-3.\theta$,

$a \cdot \cos \overline{n-2} \cdot \theta$, $a \cdot \cos \overline{n-1} \cdot \theta$, $a \cdot \cos n\theta$, $a \cdot \cos \overline{n+1} \cdot \theta$, $a \cdot \cos \overline{n+2} \cdot \theta$, &c., where n increases by unity for each successive high water. If we take the 4th, the 8th, the 12th, &c. differences of these numbers, we shall have for the differences standing opposite to $a \cdot \cos n\theta$,

$$a \cdot \cos n\theta \times 16 \sin^4 \frac{\theta}{2},$$

$$a \cdot \cos n\theta \times 256 \sin^8 \frac{\theta}{2},$$

$$a \cdot \cos n\theta \times 4096 \sin^{12} \frac{\theta}{2}.$$

Now if the inequality occupies many tides in going through its changes, that is, if θ is small, the powers of $\sin \frac{\theta}{2}$ will be very small, and these differences will therefore become smaller and smaller till they are nearly insensible. There is one value of θ , however, for which they do not become smaller, namely, that which makes $\sin \frac{\theta}{2}$ nearly = 1, or θ nearly = 180° , or in which the successive numbers $a \cdot \cos \overline{n-1} \cdot \theta$, $a \cdot \cos n\theta$, $a \cdot \cos \overline{n+1} \cdot \theta$, &c. have nearly equal magnitudes with a change of sign at every step. It is evident that this is the case of diurnal tides. Consequently, on taking the successive differences in this manner, the diurnal tide will ultimately be the only inequality sensible.

If then we stop at the fourth differences, we may say that the diurnal tide = $\frac{\text{fourth difference}}{16 \sin^4 \frac{\theta}{2}}$: if we stop at the eighth difference, the diurnal tide = $\frac{\text{eighth difference}}{256 \sin^8 \frac{\theta}{2}}$;

and so on, the expressions becoming more accurate as we advance further in the order of differences. Remarking, however, that the diurnal tide goes through all its changes in not fewer than 57 high waters, and that θ therefore differs from 180° by little more than 6° , or that $\sin \frac{\theta}{2} = \cos 3^\circ$ nearly = $1 - \frac{1}{800}$ nearly, we may consider the powers of $\sin \frac{\theta}{2}$ as equal to unity; and thus we have

$$\text{Diurnal tide} = \frac{1}{16} \times 4\text{th difference},$$

$$\text{or} = \frac{1}{256} \times 8\text{th difference},$$

&c.

The first of these formulæ was used throughout, both for heights and for times, and at both high and low waters.

Let us now consider the relation between the diurnal tide in height and that in time. Let θ be an angle increasing uniformly with the time, and increasing by 360° in a tidal day, its origin being the time of high water in the semidiurnal tide. Let a

be the diurnal tide at the first high water, b that at the first low water, c the semi-range of the semidiurnal tide. And suppose a and b to be so much smaller than c that their squares, &c. may be neglected. The height of the water above its mean height, on the law of elevation usually assumed, will be $a.\cos\theta + b.\sin\theta + c.\cos 2\theta$. This quantity will be maximum or minimum, or there will be high water or low water, when $-a.\sin\theta + b.\cos\theta - 2c.\sin 2\theta = 0$. The first approximation to the value of θ will be obtained by considering the large term only: from this we find $2c.\sin 2\theta = 0$, from which $\theta = 0$, or $=\frac{\pi}{2}$ or $=\pi$, or $=\frac{3\pi}{2}$, nearly. Substituting these values successively in the small terms, and supposing them liable to a correction x in the large term, we have,

For the first high water, $+b - 2c.\sin(0 + 2x) = 0$; or, nearly,

$$b - 4cx = 0; \text{ whence } x = \frac{b}{4c}, \text{ and } \theta = 0 + x = \frac{b}{4c}.$$

For the first low water, $-a - 2c.\sin(\pi + 2x) = 0$; or, nearly,

$$-a + 4cx = 0; \text{ whence } x = \frac{a}{4c}, \text{ and } \theta = \frac{\pi}{2} + x = \frac{\pi}{2} + \frac{a}{4c}.$$

For the second high water, $-b - 2c.\sin(2\pi + 2x) = 0$; or, nearly,

$$-b - 4cx = 0; \text{ whence } x = \frac{-b}{4c}, \text{ and } \theta = \pi + x = \pi - \frac{b}{4c}.$$

For the second low water, $+a - 2c.\sin(3\pi + 2x) = 0$; or, nearly,

$$+a + 4cx = 0; \text{ whence } x = \frac{-a}{4c}, \text{ and } \theta = \frac{3\pi}{2} - \frac{a}{4c}.$$

It appears therefore that the diurnal equation in time at the High waters of the First Division has the same sign as, and is a certain multiple of, the diurnal equation in height at the Low waters of the First Division; and that the diurnal equation in time at the Low waters of the First Division has the same relation to the diurnal equation in height at the High waters of the First Division; and similarly for those of the Second Division. The factor by which the diurnal tide at low water in height is converted into diurnal tide at high water in arc is $\frac{1}{4c}$; and, observing that π in arc corresponds to about $12^{\text{h}} 24^{\text{m}}$ in time, the factor for converting diurnal tide at low water in height into diurnal tide at high water in minutes of time is $\frac{744}{4\pi.c} = \frac{186}{\pi.c}$; and that by which the diurnal tide at low water in minutes of time is converted into diurnal tide at high water in height is $\frac{\pi.c}{186}$. The same factor applies for converting diurnal tide at high water in minutes of time into diurnal tide at low water in height. But the high and low waters of the First Division must be used together, and the high and low waters of the Second Division must be used together.

This theory cannot be expected to apply with accuracy to any place far from the

sea (as Limerick or New Ross), where the law of the height of semidiurnal tide, as depending on the time, differs sensibly from that of $\cos 2\theta$.

Upon investigating the magnitude of the diurnal tides, by the method detailed a short time since, it appears that, at most stations, the diurnal tide in height was given with great regularity; but that, at the greater number of stations, the diurnal tide in time was not very regular. In order to compare the diurnal tides by means of the theory above, as well as for the purpose of ascertaining their magnitudes with some accuracy, it was necessary so to combine them that a mean of many determinations could be made available. This was done in the following manner:—

First, it is to be remarked that in this and all the following investigations the high and low waters of the first division only are used; these being evidently sufficient for the complete solution of any problem of diurnal tides.

Next, it is well known, or may be anticipated from the investigations of the next section, that on examining successively the diurnal tides at high water (first division) on successive days, they increase, diminish, change sign, and increase and diminish with the changed sign, in nearly the same manner as the sine of an arc increasing proportionally to the time; and that the same remark applies to the diurnal tides at low water.

The first thing to be done in investigation was therefore to ascertain when the diurnal tide vanishes. This was done by taking the five diurnal tides nearest to the estimated place of evanescence and combining them by the method of minimum squares, on the supposition that the diurnal tide ought there to alter by uniform steps; an assumption sensibly correct.

The next thing was, to take the mean of all the diurnal tides between two vanishing points. Supposing them to be expressed by the law $a \cdot \sin \theta$, the mean of all these values is $\frac{\text{Sum of the values of } a \sin \theta}{\text{Number of values}}$, which is approximately expressed by $\frac{1}{\pi} a \int_0^\pi \sin \theta = \frac{2a}{\pi}$,

and hence the coefficient a , or the maximum diurnal tide, must $= \frac{\pi}{2} \times$ the mean of the diurnal tides between two vanishing points.

The following results have been obtained by these methods:—

Diurnal Tide in Height at High Water. First Division.

Approximate time at Kilbaha. 1842.	hrs.	Kilbaha.	Kilrush.	Foynes Island.	Limerick.	Casleh Bay.	Galway.	Old Head.	Mul-lagh-more.	Bun-crana.	Port Rush.	Carrow-keel.	Bally-castle.	Glen-arm.	Donagh-adee.	Ard-glass.	Clogher Head.	Kings-town.	Dun-more East.	New Ross.	Passage West.	Castle Towns-end.	
		feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
June	22.	17
	23.	18	-0.23	-0.22	-0.72	-1.05	-0.64	-0.72	-0.51	-0.76	-0.72	-0.78	-0.75	-0.63	-0.24	-0.21	+0.02	
	24.	19	-0.40	-0.29	-0.26	-0.30	-0.64	-0.73	-0.86	-0.97	-0.78	-0.83	-0.76	-0.78	-0.63	-0.59	-0.19	-0.18	-0.16	-0.02
	25.	19	-0.37	-0.26	-0.41	-0.30	-0.31	-0.34	-0.61	-0.99	-0.84	-0.81	-0.74	-0.83	-0.85	-0.76	-0.69	-0.61	-0.13	0.00	-0.16	0.00
	26.	20	-0.53	-0.50	-0.47	-0.46	-0.49	-0.42	-0.68	-0.99	-0.71	-0.58	-0.68	-0.72	-0.76	-0.74	-0.71	-0.63	-0.13	-0.16	-0.14	0.00
	27.	21	-0.30	-0.40	-0.41	-0.42	-0.38	-0.40	-0.60	-0.91	-0.62	-0.56	-0.66	-0.66	-0.71	-0.71	-0.73	-0.72	-0.14	-0.17	-0.17	-0.04
	28.	22	-0.30	-0.30	-0.36	-0.26	-0.34	-0.38	-0.40	-1.16	-0.70	-0.57	-0.63	-0.74	-0.65	-0.73	-0.79	-0.78	-0.75	-0.14	-0.18	-0.10	+0.01
	29.	22	-0.18	-0.24	-0.26	-0.24	-0.23	-0.27	-0.18	-0.88	-0.57	-0.49	-0.59	-0.53	-0.35	-0.54	-0.64	-0.67	-0.55	-0.09	-0.12	-0.02	+0.04
	30.	23	-0.09	-0.16	-0.13	-0.24	-0.12	-0.11	-0.14	-0.61	-0.64	-0.56	-0.67	-0.48	-0.32	-0.36	-0.48	-0.52	-0.39	-0.18	-0.12	-0.07	+0.01
July	2.	0	-0.01	-0.04	-0.03	-0.28	-0.02	-0.23	-0.20	-0.56	-0.54	-0.61	-0.33	-0.30	-0.22	-0.32	-0.34	-0.25	-0.14	-0.15	-0.09	-0.14
	3.	1	+0.23	+0.04	+0.05	-0.19	+0.09	-0.13	+0.09	-0.38	-0.34	-0.28	-0.18	+0.06	+0.12	+0.11	+0.05	+0.02	+0.08	-0.25	-0.13	+0.01	-0.13
	4.	2	+0.26	+0.17	+0.09	-0.02	+0.20	+0.03	+0.07	-0.24	-0.28	-0.22	-0.06	+0.08	+0.16	+0.16	+0.19	+0.17	+0.16	-0.20	-0.06	+0.05	+0.13
	5.	3	+0.21	+0.06	+0.10	+0.25	+0.23	+0.11	+0.02	+0.05	-0.16	+0.01	-0.05	+0.16	+0.12	+0.06	+0.14	+0.17	+0.15	-0.04	+0.06	+0.11	+0.13
	6.	4	+0.11	-0.12	+0.12	+0.28	+0.16	+0.19	+0.28	+0.41	+0.16	+0.13	+0.05	+0.32	+0.27	+0.22	+0.21	+0.21	+0.20	+0.01	+0.06	-0.03	+0.04
	7.	4	+0.07	+0.13	-0.22	+0.46	+0.02	+0.12	+0.21	+0.41	+0.44	+0.46	+0.32	+0.51	+0.38	+0.41	+0.37	+0.17	+0.28	+0.11	+0.08	-0.03	-0.05
	8.	5	+0.24	+0.28	+0.42	+0.64	+0.14	+0.18	+0.20	+0.31	+0.52	+0.53	+0.56	+0.51	+0.56	+0.54	+0.51	+0.47	+0.45	+0.18	+0.31	+0.02	-0.08
	9.	6	+0.37	+0.31	+0.47	+0.72	+0.46	+0.32	+0.55	+0.76	+0.74	+0.57	+0.65	+0.61	+0.79	+0.72	+0.27	+0.58	+0.51	+0.11	+0.24	+0.11	-0.06
	10.	7	+0.29	+0.21	-0.22	+0.23	+0.22	+0.20	+0.42	+0.69	+0.51	+0.27	+0.25	+0.26	+0.59	+0.50	+0.34	+0.52	+0.39	+0.03	+0.11	+0.06	+0.03
	11.	8	+0.16	+0.24	+0.23	+0.22	+0.10	+0.21	-0.34	+0.84	+0.56	+0.43	+0.37	+0.40	+0.79	+0.67	+0.41	+0.56	+0.43	+0.11	+0.22	+0.16	-0.02
	12.	9	+0.26	+0.31	+0.28	+0.16	+0.27	+0.20	+0.47	+0.83	+0.45	+0.36	+0.39	+0.41	+0.98	+0.88	+0.69	+0.75	+0.56	+0.19	+0.28	+0.16	-0.02
	13.	9	+0.11	+0.12	+0.09	-0.02	+0.04	-0.09	+0.29	+0.82	+0.50	+0.40	+0.45	+0.44	+0.80	+0.71	+0.56	+0.66	+0.42	+0.11	+0.17	+0.11	-0.01
	14.	10	+0.04	+0.09	+0.06	-0.05	-0.05	-0.08	+0.21	+0.93	+0.44	+0.28	+0.25	+0.27	+0.69	+0.62	+0.49	+0.52	+0.37	+0.16	+0.12	-0.06	-0.07
	15.	11	+0.03	-0.01	-0.08	+0.03	-0.10	-0.07	+0.15	+0.62	+0.34	+0.29	+0.16	+0.27	+0.56	+0.49	+0.48	+0.33	+0.38	+0.13	-0.03	-0.01	-0.01
	16.	12	-0.12	-0.18	-0.23	-0.17	-0.24	-0.29	-0.20	+0.27	+0.27	+0.20	+0.02	+0.29	+0.27	+0.23	+0.21	+0.28	+0.11	+0.13	+0.01	-0.01	-0.02
	17.	13	-0.27	-0.20	-0.28	-0.47	-0.30	-0.37	-0.50	-0.01	+0.02	-0.02	-0.05	+0.01	-0.04	-0.09	+0.08	-0.04	-0.09	-0.01	-0.02	-0.06	-0.04
	18.	14	-0.33	-0.32	-0.35	-0.54	-0.24	-0.38	-0.27	-0.32	-0.29	-0.23	-0.28	-0.25	-0.23	-0.24	-0.22	-0.26	-0.23	-0.09	-0.12	-0.12	-0.08
	19.	15	-0.37	-0.18	-0.41	-0.40	-0.32	-0.31	-0.37	-0.52	-0.44	-0.41	-0.48	-0.38	-0.28	-0.28	-0.42	-0.36	-0.29	-0.16	-0.31	-0.18	-0.12
	20.	16	-0.41	-0.36	-0.41	-0.38	-0.39	-0.41	-0.57	-0.74	-0.61	-0.55	-0.64	-0.50	-0.34	-0.44	-0.50	-0.50	-0.39	-0.20	-0.35	-0.19	-0.13
	21.	17	-0.42	-0.34	-0.39	-0.41	-0.41	-0.41	-0.55	-0.79	-0.68	-0.62	-0.62	-0.58	-0.43	-0.51	-0.58	-0.52	-0.46	-0.21	-0.27	-0.19	-0.10
	22.	18	-0.42	-0.39	-0.38	-0.38	-0.40	-0.39	-0.50	-0.86	-0.73	-0.64	-0.74	-0.61	-0.48	-0.52	-0.60	-0.53	-0.54	-0.28	-0.33	-0.25	-0.15
	23.	18	-0.43	-0.40	-0.42	-0.37	-0.42	-0.35	-0.50	-0.90	-0.75	-0.40	-0.71	-0.63	-0.58	-0.62	-0.62	-0.66	-0.53	-0.19	-0.29	-0.24	-0.12
	24.	19	-0.42	-0.39	-0.39	-0.42	-0.43	-0.39	-0.59	-0.96	-0.64	-0.56	-0.58	-0.59	-0.64	-0.68	-0.69	-0.83	-0.49	-0.25	-0.28	-0.19	-0.09
	25.	20	-0.37	-0.36	-0.38	-0.38	-0.38	-0.33	-0.57	-0.87	-0.57	-0.52	-0.46	-0.53	-0.65	-0.65	-0.67	-0.66	-0.50	-0.20	-0.21	-0.16	-0.10
	26.	20	-0.33	-0.36	-0.33	-0.32	-0.33	-0.10	-0.45	-0.87	-0.59	-0.47	-0.37	-0.50	-0.58	-0.60	-0.65	-0.63	-0.55	-0.16	-0.26	-0.18	-0.03
	27.	21	-0.22	-0.25	-0.24	-0.32	-0.25	-0.21	-0.45	-0.72	-0.49	-0.33	-0.37	-0.46	-0.49	-0.58	-0.56	-0.71	-0.54	-0.14	-0.23	-0.10	-0.01
	28.	22	-0.14	-0.15	-0.16	-0.24	-0.19	-0.23	-0.25	-0.55	-0.42	-0.21	-0.16	-0.33	-0.45	-0.59	-0.50	-0.51	-0.48	-0.08	-0.18	-0.03	-0.01
	29.	22	-0.09	-0.03	-0.02	-0.07	-0.10	-0.05	-0.05	-0.50	-0.39	-0.24	-0.23	-0.29	-0.22	-0.34	-0.31	-0.18	-0.22	-0.19	-0.17	-0.05	+0.04
	30.	23	+0.01	0.00	+0.02	-0.09	0.00	+0.05	0.00	-0.11	-0.37	-0.47	-0.36	-0.30	-0.11	-0.16	-0.24	-0.21	-0.17	-0.14	-0.15	-0.06	+0.01
August	1.	0	+0.11	+0.11	+0.07	+0.04	+0.14	+0.10	+0.22	+0.09	-0.30	-0.27	-0.32	-0.02	+0.16	+0.10	-0.02	-0.04	+0.03	-0.04	-0.09	-0.07	-0.01
	2.	1	+0.21	+0.08	+0.20	+0.24	+0.25	+0.18	+0.57	+0.09	+0.01	+0.08	+0.03	+0.31	+0.35	+0.30	+0.16	+0.21	+0.24	-0.06	-0.06	-0.07	-0.04
	3.	2	+0.26	+0.07	+0.14	+0.22	+0.21	+0.12	+0.54	+0.10	+0.08	+0.03	-0.03	+0.21	+0.27	+0.20	+0.20	+0.21	+0.24	+0.04	+0.02	-0.04	-0.02
	4.	3	+0.27	+0.14	+0.19	+0.22	+0.21	+0.07	+0.43	+0.29	+0.33	+0.25	+0.23	+0.31	+0.20	+0.18	+0.15	+0.19	+0.21	+0.29	+0.12	+0.11	+0.12
	5.	4	+0.20	+0.09	+0.28	+0.34	+0.26	+0.17	+0.61	+0.69	+0.57	+0.56	+0.63	+0.58	+0.56	+0.52	+0.42	+0.55	+0.49	+0.28	+0.13	+0.20	+0.20
	6.	5	+0.14	+0.11	+0.20	+0.29	+0.17	+0.14	+0.44	+0.72	+0.51	+0.44	+0.32	+0.45	+0.53	+0.53	+0.54	+0.52	+0.47	+0.15	+0.22	+0.17	+0.11
	7.	6	+0.16	+0.19	+0.29	+0.36	+0.20	+0.21	+0.54	+0.68	+0.81	+0.70	+0.64	+0.65	+0.65	+0.62	+0.66	+0.49	+0.51	+0.28	+0.24	+0.17	+0.04
	8.	6	+0.33	+0.38	+0.44	+0.49	+0.45	+0.39	+0.69	+0.96	+0.83	+0.74	+0.89	+0.72	+0.97	+0.94	+0.85	+0.65	+0.69	+0.29	+0.34	+0.23	+0.03
	9.	7	+0.31	+0.33	+0.35	+0.36	+0.34	+0.30	+0.48	+0.90	+0.51	+0.41	+0.58	+0.44	+0.81	+0.80	+0.59	+0.53	+0.54	+0.18	+0.29	+0.22	+0.03
	10.	8	+0.16	+0.16	+0.22	+0.19	+0.11	+0.10	+0.27	+0.80	+0.50	+0.41	+0.44	+0.31	+0.87	+0.77	+0.56	+0.52	+0.61	+0.16	+0.16	+0.09	-0.04
	11.	9	+0.09	+0.10	+0.10	+0.08	-0.01	+0.07	+0.11	+0.72	+0.42	+0.19	+0.15	+0.14	+0.84	+0.74	+0.71	+0.56	+0.53	+0.01	+0.12	+0.03	-0.07
	12.	10	-0.03	-0.02	+0.02	-0.09	-0.11	-0.01	+0.08	+0.47	+0.15	+0.07	-0.06	+0.14	+0.68	+0.57	+0.57	+0.38	+0.55	-0.03	-0.03	-0.06	-0.08
	13.	11	-0.13	-0.1																			

Diurnal Tide in Height at Low Water. First Division.

Approximate time at Kibbaha. 1842.		Kilbaha.	Kilrush.	Foynes Island.	Lime-rick.	Casleh Bay.	Galway.	Old Head.	Mul-lagh-more.	Bun-crana.	Port Rush.	Carrow-keel.	Bally-castle.	Glen-arm.	Donagh-adee.	Ard-glass.	Clogher Head.	Kings-town.	Dun-more East.	New Ross.	Passage West.	Castle Towns-end.
hrs.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.	feet.
June 22.	23							+0.47	+0.30	-0.30	-0.01	-0.32		+0.65	+0.63	+0.56	+0.54	+0.59	-0.18	+0.03		+0.03
23.	23															+0.40						
25.	0	+0.64			+0.71	+0.60	+0.64	+0.14	-0.21	-0.06	-0.39		+0.73	+0.72	+0.55	+0.59	+0.58	-0.15	+0.03	-0.12	+0.03	
26.	1	+0.52	+0.50	+1.13	+0.57	+0.58	+0.53	+0.15	-0.19	-0.36	-0.33		+0.43	+0.42	+0.35	+0.36	+0.32	-0.31	-0.19	-0.12	-0.03	
27.	1	+0.46	+0.44	+0.55	+0.63	+0.46	+0.49	+0.36	-0.05	-0.39	-0.41	-0.26		+0.36	+0.39	+0.38	+0.41	+0.25	-0.26	-0.10	-0.08	+0.01
28.	2	+0.49	+0.57	+0.65	+0.75	+0.55	+0.57	+0.41	+0.09	-0.33	-0.13	-0.09	+0.14	+0.40	+0.46	+0.48	+0.42	+0.04	-0.17	-0.06	-0.08	0.00
29.	3	+0.44	+0.50	+0.51	+0.61	+0.50	+0.52	+0.33	-0.16	-0.26	-0.02	-0.10	+0.14	+0.15	+0.32	+0.33	+0.37	+0.31	-0.23	-0.17	-0.04	+0.01
30.	4	+0.39	+0.40	+0.51	+0.63	+0.41	+0.52	+0.36	+0.26	-0.11	+0.06	+0.03	+0.14	+0.19	+0.30	+0.30	+0.32	+0.32	-0.11	-0.11	-0.01	+0.07
July 1.	5	+0.39	+0.41	+0.51	+0.95	+0.42	+0.45	+0.60	+0.50	+0.07	+0.14	+0.21	+0.09	+0.25	+0.34	+0.31	+0.38	+0.35	-0.11	-0.10	-0.10	-0.05
2.	6	+0.22	+0.19	+0.32	+0.47	+0.18	+0.26	+0.34	+0.41	-0.05	-0.06	0.00	-0.11	+0.19	+0.21	+0.18	+0.18	+0.22	-0.13	-0.08	-0.07	+0.04
3.	7	+0.24	+0.16	+0.23	+0.29	+0.13	+0.13	+0.25	+0.28	+0.06	+0.03	+0.04	+0.05	+0.27	+0.32	+0.28	+0.18	+0.29	+0.01	+0.07	+0.03	+0.13
4.	8	+0.20	+0.18	+0.10	-0.12	+0.16	+0.11	+0.28	+0.08	+0.03	+0.02	+0.06	+0.15	+0.43	+0.35	+0.29	-0.04	+0.16	+0.11	+0.25	+0.11	+0.06
5.	9	-0.14	-0.05	-0.21	-0.55	-0.19	-0.28	-0.04	+0.17	-0.17	-0.09	+0.01	-0.04	+0.10	+0.01	-0.11	-0.17	-0.17	+0.04	0.00	0.00	+0.08
6.	10	-0.34	-0.40	-0.49	-0.72	-0.45	-0.48	-0.26	+0.35	+0.05	-0.05	-0.01	-0.04	-0.17	-0.16	-0.16	-0.09	-0.18	0.00	-0.05	-0.04	+0.02
7.	11	-0.40	-0.60	-0.74	-1.16	-0.52	-0.61	-0.27	+0.40	+0.11	+0.11	+0.03	+0.06	-0.34	-0.39	-0.20	-0.37	-0.42	+0.05	-0.17	+0.01	-0.09
8.	12	-0.16	-0.71	-0.90	-1.37	-0.65	-0.68	-0.56	+0.09	-0.03	+0.07	-0.06	+0.02	-0.62	-0.64	-0.64	-0.64	-0.64	+0.06	-0.19	0.00	-0.27
9.	12	-0.34	-0.42	-0.61	-0.57	-0.46	-0.56	-0.62	+0.02	+0.26	+0.47	+0.20	+0.37	-0.39	-0.40	-0.38	-0.38	-0.32	+0.08	+0.03	+0.07	-0.17
10.	13	-0.23	-0.08	-0.51	-0.61	-0.46	-0.57	-0.73	-0.05	+0.12	+0.22	+0.16	+0.14	-0.51	-0.44	-0.34	-0.34	-0.27	+0.05	+0.23	+0.38	+0.03
11.	14	-0.60	-0.47	-0.58	-0.79	-0.67	-0.68	-0.82	-0.46	+0.09	+0.24	+0.17	+0.15	-0.70	-0.63	-0.58	-0.66	-0.56	+0.09	+0.06	+0.12	-0.05
12.	14	-0.44	-0.29	-0.55	-0.53	-0.50	-0.63	-0.61	-0.43	-0.09	+0.33	+0.35	+0.07	-0.57	-0.50	-0.49	-0.49	-0.42	+0.10	+0.19	+0.16	-0.03
13.	15	-0.39	-0.41	-0.55	-0.50	-0.39	-0.48	-0.55	-0.36	+0.10	+0.11	+0.25	-0.21	-0.64	-0.56	-0.58	-0.61	-0.56	+0.19	-0.10	+0.09	-0.09
14.	16	-0.27	-0.22	-0.29	-0.27	-0.27	-0.29	-0.44	-0.44	+0.09	+0.05	+0.13	-0.28	-0.39	-0.38	-0.44	-0.47	-0.44	+0.15	+0.09	+0.07	-0.25
15.	17	-0.20	-0.30	-0.16	-0.14	-0.24	-0.20	-0.37	-0.44	0.00	-0.01	+0.04	-0.30	-0.28	-0.23	-0.26	-0.14	-0.31	+0.16	+0.10	+0.11	-0.26
16.	18	-0.07	-0.09	-0.05	+0.05	-0.16	-0.39	-0.39	-0.45	+0.01	-0.10	+0.04	-0.27	-0.24	-0.19	+0.14	-0.23	-0.21	+0.06	-0.07	-0.04	-0.38
17.	20	+0.13	+0.12	+0.14	+0.14	+0.11	+0.03	-0.12	-0.20	0.00	-0.06	+0.01	-0.03	-0.01	+0.02	+0.08	-0.03	0.00	+0.01	-0.02	-0.08	-0.02
18.	21	+0.26	+0.28	+0.27	+0.26	+0.28	+0.25	+0.18	-0.19	-0.09	-0.10	+0.03	-0.08	-0.11	+0.17	+0.10	+0.11	+0.11	-0.09	-0.07	-0.02	0.00
19.	22	+0.33	+0.34	+0.36	+0.39	+0.36	+0.41	+0.33	-0.25	-0.15	-0.13	-0.04	-0.13	+0.09	+0.31	+0.23	+0.26	+0.24	-0.06	-0.14	-0.05	-0.01
20.	22	+0.41	+0.39	+0.45	+0.40	+0.45	+0.46	+0.52	-0.07	-0.14	-0.18	-0.12	-0.14	+0.42	+0.44	+0.36	+0.25	+0.32	-0.09	-0.15	-0.07	-0.02
21.	23	+0.43	+0.39	+0.46	+0.32	+0.51	+0.44	+0.56	+0.01	-0.16	-0.25	-0.21	-0.19	+0.45	+0.46	+0.36	+0.31	+0.29	-0.10	-0.11	-0.05	0.00
23.	0	+0.39	+0.35	+0.36	+0.27	+0.42	+0.33	+0.48	0.00	-0.21	-0.34	-0.24	-0.26	+0.47	+0.46	+0.38	+0.25	+0.27	-0.18	-0.18	-0.11	-0.06
24.	0	+0.41	+0.23	+0.40	+0.31	+0.45	+0.41	+0.48	-0.01	-0.28	-0.40	-0.32	-0.31	+0.44	+0.44	+0.35	+0.21	+0.31	-0.20	-0.13	-0.11	-0.01
25.	1	+0.36	+0.25	+0.46	+0.32	+0.46	+0.46	+0.47	+0.05	-0.31	-0.41	-0.34	-0.22	+0.38	+0.38	+0.30	+0.31	+0.27	-0.24	-0.15	-0.07	+0.08
26.	1	+0.33	+0.30	+0.41	+0.26	+0.45	+0.41	+0.40	+0.06	-0.24	-0.33	-0.25	-0.13	+0.31	+0.32	+0.30	+0.32	+0.27	-0.15	-0.15	-0.07	+0.04
27.	2	+0.35	+0.29	+0.38	+0.35	+0.42	+0.47	+0.34	-0.03	-0.19	-0.28	-0.25	-0.09	+0.23	+0.26	+0.28	+0.30	+0.26	-0.15	-0.14	-0.11	0.00
28.	2	+0.31	+0.36	+0.37	+0.55	+0.41	+0.52	+0.39	+0.04	-0.14	-0.23	-0.23	-0.04	+0.26	+0.28	+0.31	+0.35	+0.25	-0.28	-0.17	-0.10	-0.03
29.	3	+0.28	+0.31	+0.30	+0.39	+0.28	+0.41	+0.35	-0.05	-0.19	-0.18	-0.12	+0.02	+0.19	+0.19	+0.26	+0.28	+0.27	-0.15	-0.18	-0.10	+0.02
30.	4	+0.27	+0.21	+0.32	+0.48	+0.23	+0.29	+0.39	+0.21	+0.04	+0.04	+0.11	+0.11	+0.20	+0.19	+0.27	+0.31	+0.27	-0.06	-0.05	-0.01	-0.05
31.	5	+0.19	+0.15	+0.28	+0.41	+0.18	+0.17	+0.43	+0.34	+0.15	+0.07	+0.22	0.00	+0.09	+0.18	+0.25	+0.27	+0.19	-0.04	+0.01	0.00	-0.11
August 1.	6	-0.02	+0.07	+0.09	+0.10	+0.18	0.00	-0.21	+0.32	+0.11	+0.01	-0.11	+0.14	+0.17	+0.11	+0.12	+0.12	+0.11	+0.06	+0.05	0.00	-0.03
2.	8	-0.15	-0.08	-0.09	-0.21	-0.14	-0.08	+0.01	+0.27	+0.16	+0.06	+0.02	+0.02	+0.25	+0.16	+0.10	+0.23	+0.06	+0.09	+0.09	0.00	0.00
3.	9	-0.31	-0.29	-0.30	-0.48	-0.22	-0.16	-0.12	+0.14	+0.03	-0.07	-0.04	0.00	+0.12	+0.02	-0.12	+0.26	-0.21	+0.01	-0.04	-0.03	-0.04
4.	9	-0.37	-0.39	-0.44	-0.75	-0.33	-0.02	-0.36	-0.01	-0.01	-0.09	+0.12	-0.10	-0.26	-0.35	-0.40	-0.36	-0.39	-0.03	-0.09	-0.02	-0.02
5.	10	-0.51	-0.45	-0.60	-0.70	-0.58	0.00	-0.37	-0.04	+0.11	+0.19	+0.26	+0.06	-0.55	-0.58	-0.50	-0.42	-0.43	+0.11	-0.04	+0.08	+0.01
6.	11	-0.45	-0.32	-0.57	-0.47	-0.49	-0.42	-0.49	+0.18	0.00	+0.08	+0.11	+0.08	-0.71	-0.66	-0.62	-0.69	-0.50	+0.08	-0.01	+0.08	-0.01
7.	12	-0.71	-0.52	-0.68	-0.35	-0.60	-0.64	-0.72	-0.23	-0.03	+0.27	+0.20	+0.15	-1.04	-0.88	-0.82	-0.64	-0.64	+0.06	-0.09	+0.06	-0.06
8.	12	-0.48	-0.51	-0.49	-0.11	-0.61	-0.42	-0.61	-0.17	+0.34	+0.55	+0.47	+0.50	-0.66	-0.58	-0.55	-0.40	-0.39	+0.04	+0.13	+0.02	-0.06
9.	13	-0.34	-0.34	-0.34	-0.16	-0.40	-0.41	-0.63	-0.03	+0.26	+0.44	+0.29	+0.26	-0.53	-0.46	-0.45	-0.41	-0.41	+0.08	+0.12	+0.07	-0.04
10.	14	-0.28	-0.30	-0.28	-0.12	-0.29	-0.26	-0.57	+0.02	+0.33	+0.26	+0.31	+0.13	-0.42	-0.34	-0.36	-0.36	-0.38	+0.11	+0.08	+0.07	-0.09
11.	14	-0.15	-0.24	-0.18	-0.18	-0.15	-0.08	-0.57	+0.10	+0.37	+0.40	+0.36	+0.07	-0.23	-0.19	-0.19	-0.17	-0.17	+0.24	+0.10	+0.09	-0.17
12.	15	-0.04	-0.14	-0.09	-0.19	-0.05	-0.20	-0.51	-0.09	+0.27	+0.27	+0.16	-0.03	-0.09	-0.07	-0.09	-0.13	-0.14	+0.20	+0.04	+0.06	-0.06
13.	16	+0.05	-0.08	+0.04	+0.05	+0.06	+0.10	-0.27	-0.07	+0.09	+0.03	+0.01	-0.03	-0.02	+0.03	-0.01	-0.03	-0.06	+0.10	+0.04	+0.04	+0.04
14.	18	+0.17	+0.18	+0.26	+0.40	+0.21	+0.25	+0.04	-0.04	-0.02	-0.02	-0.01	+0									

Times of Evanescence of Diurnal Tide in Height.

Kilbaha	High water.	d. July 1·73	d. July 14·97	d. July 30·72	d. Aug. 12·03
	Low water.	4·59	17·01	31·84	13·15
Kilrush	High water.	(1·8)	15·17	31·46	12·27
	Low water.	4·30	17·03	32·29	13·68
Foynes Island	High water.	1·98	14·74	30·74	12·31
	Low water.	4·27	16·98	32·64	13·19
Limerick	High water.	(1·9)	(14·7)	31·30	11·86
	Low water.	4·00	16·86	32·54	13·05
Casleh Bay	High water.	2·34	13·98	30·82	11·55
	Low water.	4·10	17·21	32·41	13·10
Galway	High water.	3·83	13·71	30·77	11·99
	Low water.	4·24	17·50	32·56	13·04
Old Head	High water.	(4·8)	15·62	31·36	12·38
	Low water.	5·22	18·11	33·46	14·80
Mullaghmore	High water.	4·61	17·63	32·53	14·17
	Low water.	(7·0)	(20·0)	(35·0)	(16·0)
Buncrana	High water.	5·38	17·56	33·32	13·74
	Low water.	(7·4)	(20·0)	(35·0)	15·63
Port Rush	High water.	5·09	17·26	33·43	13·47
	Low water.	(7·4)	(20·0)	(35·0)	14·92
Carrowkeel	High water.	4·96	16·66	33·26	12·69
	Low water.	(7·4)	(20·0)	(35·0)	(14·9)
Ballycastle	High water.	3·80	17·68	32·37	14·51
	Low water.	(6·4)	(19·0)	(34·0)	(13·9)
Glenarm	High water.	2·80	18·11	31·72	15·62
	Low water.	5·92	18·78	34·24	13·84
Donaghadee	High water.	2·82	17·91	31·85	15·35
	Low water.	5·34	17·92	33·18	13·56
Ardglass	High water.	3·76	18·00	32·82	15·10
	Low water.	5·57	16·11	33·47	13·71
Clogher Head	High water.	3·51	17·46	32·48	15·05
	Low water.	4·82	18·23	34·34	14·60
Kingstown	High water.	3·70	17·62	32·22	15·44
	Low water.	5·10	18·41	33·25	14·16
Dunmore East	High water.	6·01	17·44	33·40	12·39
	Low water.	3·56	18·31	32·13	15·88
New Ross	High water.	(6·0)	(17·4)	33·57	12·95
	Low water.	(3·5)	(18·3)	(32·1)	(15·8)
Passage West	High water.	(5·0)	(16·4)	33·76	12·27
	Low water.	(2·5)	16·91	(31·1)	14·75
Castle Townsend	High water.	(3·4)	(15·7)	(32·24)	(12·15)
	Low water.	(3·5)	(17·0)	(31·5)	(13·95)

The numbers inclosed in brackets are supplied by conjecture, where the irregularity of the tides made it difficult to discover with accuracy the times of evanescence. A small error in these will produce very little error in the result for maximum coefficient. The numbers at Castle Townsend (where the diurnal tide is very small) are the means between those for the adjacent stations, Passage West and Kilbaha. Courtown is omitted, as the peculiarity of the tides there made it impossible to take diurnal tides at high water and low water in the same manner as for the other stations.

The times when the moon's declination vanished are June 28^d.89, July 12^d.14, July 26^d.18, and August 8^d.50. In stating these, however, I must warn the reader that these are not the only elements on which the time of evanescence of diurnal tide depends, as will appear in the next section.

Attempts were made to determine, in the same manner, the times of evanescence of diurnal tide in time. The irregularities were however so great that in most cases it was useless to attempt to assign the day: in the following instances only did the results appear at all trustworthy.

Times of Evanescence of Diurnal Tide in Time.

Mullaghmore	Low water.	July 7 ^d .19	July 20 ^d .76	July 31 ^d .96	Aug. 14 ^d .27
Buncrana	Low water.	3.30	20.41	30.77	16.02
Port Rush	Low water.	4.69	21.32	31.32	16.58
Carrowkeel	Low water.	1.37	19.01	30.50	16.39
Ballycastle	Low water.	5.11	18.16	36.43	15.05
Donaghadee	High water.	4.11	19.07	32.59	16.18
	Low water.	6.13	14.18	33.94	13.15

It will be remembered that the time of evanescence of diurnal tide in time at low water ought to coincide with that for height at high water, and *vice versa*. The comparison of this table with that for height is not very satisfactory.

The maximum values of diurnal tide were deduced in all cases by the process explained a short time since, adopting for the times of evanescence the days given in the first table (or that from heights), and using the high water evanescence in height with the low water diurnal tide in time, and *vice versa*. The following are the results:—

Maximum Values of Diurnal Tide, First Division: deduced from heights.

	High water.			Low water.		
	July 1—14.	July 14—31.	July 31—Aug. 12.	July 4—17.	July 17—32.	Aug. 1—13.
	ft.	ft.	ft.	ft.	ft.	ft.
Kilbaha	+0.28	-0.49	+0.30	-0.47	+0.52	-0.50
Kilrush	+0.20	-0.38	+0.25	-0.52	+0.42	-0.47
Foynes Island.	+0.30	-0.45	+0.33	-0.74	+0.52	-0.58
Limerick.	+0.30	-0.49	+0.36	-0.96	+0.49	-0.53
Casleh Bay	+0.27	-0.41	+0.30	-0.64	+0.52	-0.55
Galway	+0.24	-0.42	+0.25	-0.77	+0.57	-0.35
Old Head	+0.46	-0.61	+0.64	-0.69	+0.58	-0.74
Mullaghmore	+0.83	-0.80	+0.85	-0.30	+0.14	-0.08
Buncrana	+0.64	-0.75	+0.66	+0.05	-0.16	+0.25
Port Rush	+0.50	-0.57	+0.60	+0.14	-0.28	+0.35
Carrowkeel.	+0.44	-0.93	+0.60	+0.17	-0.19	+0.33
Ballycastle	+0.50	-0.66	+0.58	+0.05	-0.16	+0.16
Glenarm	+0.74	-0.66	+0.82	-0.60	+0.42	-0.55
Donaghadee	+0.64	-0.74	+0.74	-0.64	+0.41	-0.71
Ardglass.	+0.55	-0.74	+0.69	-0.60	+0.38	-0.58
Clogher Head	+0.64	-0.69	+0.60	-0.55	+0.36	-0.57
Kingstown	+0.53	-0.57	+0.64	-0.55	+0.36	-0.52
Dunmore East	+0.17	-0.24	+0.25	+0.13	-0.22	+0.11
New Ross	+0.22	-0.33	+0.25	+0.05	-0.22	+0.05
Passage West.	+0.08	-0.20	+0.17	+0.13	-0.11	+0.06
Castle Townsend.	-0.02	-0.09	+0.05	-0.16	-0.01	-0.08

These results are on the whole satisfactory. There are, however, some general differences of magnitude among the different columns, which I am not able at present entirely to explain. I may remark that the moon was in perigee on July 9 and August 7, and in apogee on July 25.

Maximum Coefficient of Diurnal Tide in Time, First Division.

	Low water.			High water.		
	July 1—14.	July 14—31.	July 31—Aug. 12.	July 4—17.	July 17—32.	Aug. 1—13.
	m	m	m	m	m	m
Kilbaha	+ 5.31	- 2.69	+ 4.25	- 6.22	+ 3.70	- 0.98
Kilrush	+ 1.34	- 1.87	+ 1.75	- 5.92	+ 3.70	- 3.18
Foynes Island.	+ 4.10	- 4.00	+ 5.11	- 5.63	+ 1.70	- 3.84
Limerick.	+12.23	- 8.86	+ 8.15	- 3.91	+ 0.69	- 5.63
Casleh Bay	+ 2.51	- 2.40	+ 3.18	- 8.73	+ 6.06	- 4.29
Galway	+ 5.03	- 6.97	+ 3.08	- 8.73	+ 3.64	- 4.98
Old Head	+ 2.87	- 2.96	+ 0.66	- 4.32	+ 4.13	+ 0.26
Mullaghmore	+ 3.93	- 7.77	+10.56	-10.48	+ 2.06	- 5.09
Buncrana	+10.90	- 1.51	+ 7.90	- 0.42	+ 0.34	- 1.73
Port Rush	+21.43	- 4.85	+16.77	- 1.88	+ 7.28	- 3.14
Carrowkeel.	+17.93	- 3.35	+14.92	- 5.60	+ 2.71	- 0.68
Ballycastle	+28.28	-31.00	+17.26	- 3.42	+18.05	-1.96
Glenarm	- 1.76	- 7.82	+ 4.09	+ 0.66	+ 2.68	- 2.20
Donaghadee	+ 0.04	- 6.58	+ 0.57	- 5.27	+ 1.72	- 4.52
Ardglass.	+ 1.72	- 4.41	+ 2.70	+ 0.07	- 4.47	- 0.35
Clogher Head	+ 3.80	- 5.10	+ 1.86	- 0.01	- 2.47	- 3.10
Kingstown	+ 4.32	- 5.46	+ 5.65	- 4.35	+ 0.52	- 3.92
Dunmore East	+ 0.54	- 2.77	+ 3.31	+ 3.86	- 0.24	- 2.40
New Ross	- 1.76	- 0.48	+ 3.45	+ 2.54	- 2.05	- 4.99
Passage West.	+ 2.71	+ 2.68	+ 0.96	+ 0.23	+ 0.70	- 0.51
Castle Townsend.	- 6.57	+ 2.12	- 1.26	- 2.49	+ 1.37	- 3.67

After the statement which I have given (in the second section) of the difficulty of fixing upon times of high and low water, it will not be surprising that considerable irregularities exist among these numbers. Their agreement nevertheless is sufficient to show that the diurnal tide in time of low water is great at Limerick, and very great at all the stations from Buncrana to Ballycastle. At Mullaghmore and Ballycastle it is also great at high water. The increase in numbers at low water from Kilrush to Foynes Island and Limerick, would seem to show that diurnal tide in time at low water increases considerably in ascending a river. It would appear, however (as seems, *à priori*, probable), that this holds only when there is at the same time a considerable diurnal tide in height, of such a nature that a depression of height accompanies a retardation of time. This is supported entirely by the analogy of the course of low water at ordinary semidiurnal tide: where, as will appear in this paper, and as is known from other observations, and as also appears from theory*, the progress of the phase of low water up a river is slower as the water is shallower at low water. At New Ross, considered with relation to Dunmore East, the diurnal tide in time of low water is not sensibly increased; and here there is no large diurnal tide in height. The large numbers in the neighbourhood of Ballycastle do not depend on this cause.

Maximum Coefficient of Diurnal Tide in Height, First Division, as inferred from Diurnal Tide in Time.

	High water (from times of low water).			Low water (from times of high water).		
	July 1—14.	July 14—31.	July 31—Aug. 12.	July 4—17.	July 17—32.	Aug. 1—13.
	ft.	ft.	ft.	ft.	ft.	ft.
Kilbaha	+0.46	-0.18	+0.37	-0.59	+0.28	-0.13
Kilrush	+0.11	-0.14	+0.12	-0.62	+0.30	-0.37
Foynes Island.	+0.41	-0.40	+0.61	-0.68	+0.19	-0.50
Limerick.	+1.88	-1.16	+1.35	-0.61	+0.06	-0.81
Castle Bay	+0.24	-0.16	+0.30	-0.92	+0.46	-0.45
Galway	+0.60	-0.51	+0.45	-0.87	+0.28	-0.68
Old Head	+0.28	-0.19	+0.07	-0.40	+0.24	+0.02
Mullaghmore	+0.16	-0.42	+1.02	-0.78	+0.17	-0.33
Buncrana	+0.97	-0.15	+0.75	-0.15	+0.04	-0.25
Port Rush	+0.77	-0.13	+0.73	-0.14	+0.14	-0.13
Carrowkeel	+0.97	-0.20	+0.85	-0.25	+0.11	-0.08
Ballycastle	+0.75	-0.42	+0.50	-0.11	+0.28	-0.08
Glenarm	-0.07	-0.31	+0.25	+0.05	+0.13	-0.13
Donaghadee	+0.06	-0.46	+0.10	-0.39	+0.11	-0.45
Ardglass.	+0.27	-0.40	+0.33	-0.04	+0.37	-0.11
Clogher Head	+0.45	-0.45	+0.32	-0.03	-0.19	-0.41
Kingstown	+0.44	-0.40	+0.57	-0.39	+0.46	-0.38
Dunmore East	+0.04	-0.20	+0.34	+0.31	-0.02	-0.20
New Ross	-0.19	-0.06	+0.37	+0.26	-0.20	-0.43
Passage West.	+0.26	+0.17	+0.09	+0.02	+0.08	-0.04
Castle Townsend..	-0.45	+0.12	-0.15	-0.17	+0.09	-0.24

These numbers ought, upon the theoretical expressions for the tides given in an earlier part of this section, to agree with the numbers in the first table in page 19.

* Encycl. Metropol., *Tides and Waves*, Art. 208.

Upon comparing them it appears that there is a very good agreement of the numbers of the littoral stations, at both high and low waters, as far as Mullaghmore or even Buncrana; and, for high water only, as far as Ballycastle. There is then great discordance till we arrive nearly at Kingstown; in a short time after this the diurnal tide becomes so small that we are less surprised at apparent discordances. From the number of the instances in which the agreement is, upon the whole, pretty good, I form my opinion that the discordance between Buncrana and Kingstown is not accidental. I have little doubt that in this channel between Ireland and Scotland (which every accurate determination shows to be a critical part for the tides), the law of diurnal tide assumes a form differing much from that supposed in the investigation. It is, however, practically almost impossible to trace this law from observations.

The results for diurnal tide used in the subsequent investigations are those in the table of pages 15 and 16 deduced from observed heights only.

Section IV.—*Theory of Diurnal Tide as referred to the actions of the Sun and Moon.*

The present section will contain little more than the account of a series of failures of investigations. But the examination of these is usually so instructive that I think it desirable to state the heads of each of the unsuccessful attempts.

In order to explain the theoretical difficulties of this investigation, the following remarks may not perhaps be misplaced.

It is not possible to separate the effects of the sun and moon by comparison of a mass of observed diurnal tides near one solstice with a similar mass at the opposite solstice. For, although (in consequence of the opposite state of the moon's declination at a given phase of the lunation) the lunar diurnal tide is different in sign, yet the solar diurnal tide is also different in sign; and thus the two diurnal tides are mingled in the same degree at both solstices. The same applies if the observations are at any opposite seasons of the year.

It is possible to separate the two effects by comparing diurnal tides near a solstice with diurnal tides near an equinox; as, in the latter, the solar diurnal tide vanishes.

Generally, it is possible to separate them by comparing two masses of diurnal tides observed at intervals of three months; as for the high (or low) waters corresponding to a given right ascension and declination of the moon in the two masses, the sun will have widely different positions in hour-angle, and therefore its effects at those two instants will be widely different.

The proportion of the effects of the sun and the moon cannot be ascertained from a single series of observations, extending through a period so short that the sun's position may be considered invariable. This will be shown by showing that the two effects, of the sun and of the moon, in producing diurnal tide at high water, follow sensibly the same law, and when added together give a compound effect following the same law. Thus: the time of high water bears a nearly invariable relation to the

time of moon's transit; and therefore, at high water, the lunar diurnal tide is always in nearly the same phase, and has no variation except from the variation of its coefficient. The magnitude of the diurnal tide at semidiurnal high water may therefore be represented by coefficient $\times \sin \beta$; and that at semidiurnal low water by coefficient $\times \cos \beta$; where β is constant. This coefficient is proportional to the sine of the moon's declination at some time previous, or (nearly) proportional to the sine of the moon's right ascension for some time previous, or to the sine of the moon's hour-angle from the sun altered by a constant. For the solar diurnal tide, the coefficient is constant, but the phase varies every day. As the time of high water bears a nearly invariable relation to the time of moon's transit, the phase of solar diurnal tide at high water must depend upon the moon's hour-angle from the sun altered by a constant, and therefore the magnitude of solar diurnal tide will be proportional to the sine of the moon's hour-angle from the sun altered by a constant. Thus, putting $\epsilon - \odot$ for the excess of the moon's right ascension above the sun's, the lunar diurnal tide at the time of high water will be represented by $a \cdot \sin \beta \cdot \sin \{ \epsilon - \odot + A \}$, and the solar diurnal tide at the same time will be represented by $b \cdot \sin \{ \epsilon - \odot + B \}$; and these, when added together, give a result of the same form, $c \cdot \sin \{ \epsilon - \odot + C \}$. And it is impossible to say whether this term, as given by observation, is entirely due to one or other of the two actions or to both combined; because we have no *à priori* means of saying what is the coefficient a or b of either of the separate terms; or what is the relation of the time of either high diurnal tide to the time of transit of the body which causes it, upon which A and B will depend. Everything here said with regard to semidiurnal high water applies also to semidiurnal low water; the only difference being that the angles β and B must be increased about 90° for semidiurnal low water.

The unknown quantities in the problem of diurnal tide are the following:—The interval anterior to the time of observation for which the moon's place is to be taken as governing the diurnal tide at the time of observation; the constant coefficient by which the sine of moon's declination for that anterior time is to be multiplied; the moon's hour-angle at the time of lunar diurnal high water; and the three similar quantities for the sun: in all, six unknown quantities. To determine these we have only the four following results of observation (or results equivalent to these four): the time of evanescence of diurnal tide at semidiurnal high water; the maximum of diurnal tide in high water, and the two similar quantities for low water. These are insufficient for the determination of the six unknown quantities; and we must try how we can reduce the latter number.

First, as the sun's declination is considered constant, the anterior interval for the sun's place is unimportant. And in fact, though the sun's declination during these observations (June 22 to August 25) was not invariable, yet an alteration of one day in the time for which its declination was taken as ruling the diurnal tide would not have been important. For the moon it would be very important.

Secondly, it seems probable that the moon's hour-angle at the time of lunar diurnal

high water does not differ much from the sun's hour-angle at the time of solar diurnal high water. The assumption of any constant difference, either =0 or having any assigned magnitude, reduces two of the unknown quantities to one.

The number of unknown quantities is thus made the same as the number of data, and the solution can therefore (speaking in a strictly algebraical sense) be effected, in general.

The following is the method by which the equations for the four unknown quantities may most conveniently be formed:—

From the ordinary facts of the tides, it seems probable that the coefficient of lunar diurnal tide may depend on the moon's declination at a few days, perhaps not exceeding five, anterior to the time of the tide. Let d_1 be the moon's declination at one day preceding the time of tide, d_3 the moon's declination at three days preceding the time of tide. Then we may express the coefficient of lunar diurnal tide by $p \cdot \sin d_1 + q \cdot \sin d_3$; where by varying the proportions of p and q the coefficient may be made to depend on the moon's declination at any day near them; and by varying the magnitudes of p and q in the same proportion, the magnitude of the coefficient will be altered in that proportion.

The coefficient of solar diurnal tide may be represented with sufficient accuracy by $r \cdot \sin D_1$, where D_1 is the sun's declination one day preceding the time of tide.

Let h be the solar hour of the tide. This is the same as the hour-angle of the sun to the west of the meridian. The phase of the solar diurnal tide will depend upon this angle diminished by some unknown constant s ; and the elevation of the solar diurnal tide may be represented by $S \cdot \sin D_1 \cdot \cos \overline{h-s}$.

Let t be the moon's time of transit. Then the moon's hour-angle west of the meridian is $h-t$. Therefore if the phase of lunar diurnal tide depended on the moon's hour-angle in the same manner in which the phase of solar diurnal tide depended on the sun's hour-angle, the elevation of the lunar diurnal tide would be represented by $(p \cdot \sin d_1 + q \cdot \sin d_3) \cos \overline{h-t-s}$. But we know by the retardation of the period of spring tides, as well as by the theory of tidal waves affected by friction*, that in semidiurnal tides the lunar wave is more advanced in its phase with regard to the moon's hour-angle than the solar wave is with regard to the sun's hour-angle. We may conjecture, by analogy, that the same holds for diurnal tide. Putting α for this difference of advance of phase, the elevation of the lunar diurnal tide will be represented by $(p \cdot \sin d_1 + q \cdot \sin d_3) \cos \overline{h-t-s+\alpha}$. And the compound effect of lunar and solar diurnal tides, expanding the cosines, will be

$$S \cdot \sin D_1 \cdot (\cos h \cdot \cos s + \sin h \cdot \sin s) \\ + (p \cdot \sin d_1 + q \cdot \sin d_3) (\cos \overline{h-t+\alpha} \cdot \cos s + \sin \overline{h-t+\alpha} \cdot \sin s).$$

Let $S \cdot \cos s = w$, $S \cdot \sin s = x$, $\frac{p}{S} = y$, $\frac{q}{S} = z$; and the expression becomes

* Encyclopædia Metropolitana, *Tides and Waves*, Art. 326.

$$\sin D_1 \cos h.w + \sin D_1 \sin h.x + \sin d_1 \cos \overline{h-t+\alpha}.wy + \sin d_1 \sin \overline{h-t+\alpha}.xy \\ + \sin d_3 \cos \overline{h-t+\alpha}.wz + \sin d_3 \sin \overline{h-t+\alpha}.xz.$$

It is to be remarked that, when w , x , y , and z , are ascertained (with an assumed value of α), the following more intelligible results will be extracted from them:—

$S = \sqrt{w^2 + x^2}$ = solar coefficient of the sine of the sun's declination, for solar diurnal tide.

s = the angle determined by the equation $\tan s = \frac{x}{w}$; it is the constant angle which is to be subtracted from the sun's hour-angle west at the time of observation, in order to give the angle on whose cosine depends the height of solar diurnal tide at the instant of observation.

Then, the lunar diurnal tide

$$= (p \cdot \sin d_1 + q \cdot \sin d_3) \cos \overline{h-t-s+\alpha} = S \cdot (y \cdot \sin d_1 + z \cdot \sin d_3) \cdot \cos \overline{h-t-s+\alpha};$$

and, {putting l for the moon's longitude measured from the intersection of its orbit with the equator, I for the sine of its inclination, and δ for the mean daily increase of longitude from transit to transit = $13^\circ 38'$ }, $y \cdot \sin d_1 + z \cdot \sin d_3 = I(y \cdot \sin l_1 + z \cdot \sin l_3)$

$$= I \left(\frac{z+y}{2} \cdot \overline{\sin l_3 + \sin l_1} + \frac{z-y}{2} \cdot \overline{\sin l_3 - \sin l_1} \right) = I(\overline{z+y} \cdot \cos \delta \cdot \sin l_2 + \overline{z-y} \cdot \sin \delta \cdot \cos l_2); \text{ or,}$$

if $\tan \eta = \frac{z-y}{z+y} \tan \delta$, this quantity becomes $= I \cdot \overline{z+y} \cdot \cos \delta \cdot \sec \eta \cdot \sin \overline{l_2 + \eta}$; or, making $\eta = n \cdot \delta$, it becomes $= I \cdot \overline{z+y} \cdot \cos \delta \cdot \sec \eta \cdot \sin l_{2+n} = \overline{z+y} \cdot \cos \delta \cdot \sec \eta \cdot \sin d_{2+n}$: and therefore the lunar diurnal tide $= S \cdot \overline{z+y} \cdot \cos \delta \cdot \sec \eta \cdot \sin d_{2+n} \cdot \cos \overline{h-t-s+\alpha}$.

$$\frac{M}{S} = \frac{\text{Effect of moon for given declination}}{\text{Effect of sun for same declination}} = \overline{z+y} \cdot \cos \delta \cdot \sec \eta.$$

$M = S \cdot \overline{z+y} \cdot \cos \delta \cdot \sec \eta$ = lunar coefficient of the sine of the moon's declination on a certain anterior day, for lunar diurnal tide.

$2+n$ = the time, in lunar days, earlier than the moon's Greenwich transit next preceding, for which the moon's declination is to be taken as governing the diurnal tide. This is correct for the time of high water, first division, and requires an alteration for other times.

n is $= \frac{\eta}{13^\circ 38'}$; and $\tan \eta = \frac{z-y}{z+y} \cdot \tan 13^\circ 38'$.

$s - \alpha$ = the constant angle which is to be subtracted from the moon's hour-angle, in the same manner as s from the sun's hour-angle.

The factors of the unknown terms w , x , wy , wz , xy , and xz , in the algebraical expression for the elevation produced by diurnal tide, were computed for high water and low water, first division, at Kilbaha, for every day throughout the observations. These computations would apply equally to the other stations, it being understood that certain constants (which the reader will easily investigate) depending on the longitude of the station and the time occupied by the passage of semidiurnal tide, are to be applied to the angles α and s . The hour-angles used for the moon were found by comparing the moon's time of transit at Greenwich with the time of Kil-

baha tide. The declinations were those for the times of transit at Greenwich one day and three days previous to the transit next preceding the tide in question. For the low waters of the first division, which follow the high waters of the first division by $\frac{1}{4}$ th of a tidal day, the right ascensions and declinations ought to be taken for transit over the 6^h meridian; this was done most conveniently by correcting the coefficients when combined in groups, by the following formulæ. If the computed terms containing w and x for the moon are $W.w + X.x$ { W and X containing y and z }, then the corrected terms as altered for the change of right ascension, are

$$\left(W \cos \frac{\delta}{4} + X \sin \frac{\delta}{4}\right)w + \left(X \cos \frac{\delta}{4} - W \sin \frac{\delta}{4}\right)x.$$

And if the computed terms containing y and z are $Y.y + Z.z$ (Y and Z containing w and x), then the corrected terms as altered for the change of declination, are

$$\left(Y \frac{\sin\left(2\delta + \frac{\delta}{4}\right)}{\sin 2\delta} - Z \frac{\sin \frac{\delta}{4}}{\sin 2\delta}\right)y + \left(Z \frac{\sin\left(2\delta - \frac{\delta}{4}\right)}{\sin 2\delta} + Y \frac{\sin \frac{\delta}{4}}{\sin 2\delta}\right)z.$$

No notice was taken of the changes of parallax; nor were the hour-angles referred to the moon's place one or three days previous (as in strictness of theory they ought); but as the observations extend over two whole lunations, it was supposed that the effects of these omissions would nearly disappear.

The factors of the unknown quantities were computed on the supposition that $\alpha=0$, and also on two other suppositions. It is easily seen, however, that the factors for any assumed value of α can be readily formed from those which hold for $\alpha=0$; and this computation is made most conveniently for the groups.

The numbers for high water were divided into groups related to the changes of sign of the factors of w and x . These groups were then combined in the order 1st + 2nd - 3rd - 4th + 5th + 6th - &c. to form one equation, and in the order 1st - 2nd - 3rd + 4th + 5th - &c. to form another equation. The numbers for low water were treated in the same way. In subsequent operations, the groups were formed and combined in different orders.

But, in whatever way the groups were formed, they were so combined as to form four equations, each of which has the following form:

$$A.w + B.x + C.wy + D.wz + E.xy + F.xz = G.$$

To solve a system of four such equations is evidently no easy matter. Two methods of solution were principally relied on.

The first (and easiest) was, to make trial-substitutions to a great extent. The numbers -2, -1, 0, +1, +2, were substituted for w ; the same numbers were substituted for x ; the same numbers were also substituted for y and for z ; and every possible combination of these numbers was used; making 625 trial-substitutions in each of the four equations. And when there seemed a probability of success, the substitutions for one or two of the numbers were greatly extended. Calling the re-

sults of one substitution in the four equations g, g', g'', g''' (the numbers resulting from the tidal observations being G, G', G'', G'''), it was then necessary that

$$\frac{g'}{g} = \frac{G'}{G}, \quad \frac{g''}{g} = \frac{G''}{G}, \quad \frac{g'''}{g} = \frac{G'''}{G}.$$

By search among the quotients of the substitution, numbers were found approaching as near as possible to $\frac{G'}{G}$, &c.; then, supposing w unaltered, the variations of the quotients were found, which corresponded to changes of 1 in x, y and z ; from these, by solving three linear equations, the corrections to x, y and z were found; and then a common multiplier for w and x was found by comparing the result of each corrected substitution with the tidal numbers $G, G',$ &c.

The other method was, to put the equations in the following form:

$$w \times (A + C.y + D.z) + x \times (B + E.y + F.z) = G.$$

Between two of the equations, w and x were eliminated, and a complicated equation between y and z remained; another equation of the same character was obtained from the other two of the original equations; and these two equations were solved by trials.

By these methods (but principally by the former) the equations were solved for $\alpha=0$ and $\alpha=-2^h$ for all the stations as far as Mullaghmore. Beyond that station it was found totally impracticable to solve them. Values of w, x, y, z were sometimes found which seemed nearly to satisfy the equations, but when an attempt was made to correct these values, the corrections became absurdly large, and the corrected values gave results much further from the truth than the original results. And for those stations at which the operation was successful, there were special results of inadmissible character. Thus, when $\alpha=0$, $\frac{M}{S}$ was found $=4.30$ for Kilbaha, and $=1.45$ for Mullaghmore; when $\alpha=-2^h$, $\frac{M}{S}$ was found $=3.40$ for Kilbaha and $=0.82$ for Mullaghmore. These discordances seemed to show that α must be positive; but in no case could a solution be obtained with a positive value for α .

On examining carefully the numbers given by observation, I was led to the following considerations, which seemed likely to throw considerable light on the subject.

On inspecting the table in page 17, it will be evident that at the first stations, as far as Old Head, the disappearance of diurnal tide at high water does not occur on the same day as the disappearance of diurnal tide at low water; the former always occurring earlier than the latter. But at the stations from Glenarm to Dunmore East, the disappearance of diurnal tide at high water sometimes precedes and sometimes follows that at low water; and may be said, roughly speaking, to occur on the same day. This circumstance fixes absolutely the value of α . For, when the diurnal tide at high water and that at low water vanish at the same time, the inference is, that at that time the lunar diurnal tide and the solar diurnal tide have equal values

with opposite signs four times in their diurnal period. If then their phases at the first of those four times be represented by ϕ and ψ respectively, and their coefficients by μ and ν , we have

$$\begin{aligned} \mu \sin \phi + \nu \sin \psi &= 0, & \mu \sin (\phi + 90^\circ) + \nu \sin (\psi + 90^\circ) &= 0, \\ \mu \sin (\phi + 180^\circ) + \nu \sin (\psi + 180^\circ) &= 0, & \mu \sin (\phi + 270^\circ) + \nu \sin (\psi + 270^\circ) &= 0. \end{aligned}$$

The solution of these equations is either $\mu = -\nu$, $\phi = \psi$; or $\mu = \nu$, $\phi = \psi + 180^\circ$. Confining our expressions to the former (by which we lose nothing of generality), we have this result; that, upon that day, the solar diurnal tide and the lunar diurnal tide are in the same phases at every part of the day. Observing then that α is the angle which is to be added to the moon's hour-angle west, in order to give it the same relation to the phase of lunar diurnal tide which the sun's hour-angle west has to the phase of solar diurnal tide, it will be seen that α must be equal to the excess of the moon's right ascension over the sun's right ascension (altered by 12^h if necessary) on the day on which the diurnal tide vanishes both at high and low water.

In order to investigate the value of α with accuracy, the following process was used:—If from the table in page 17 we form the numbers “day of evanescence at low water — day of evanescence at high water,” at Glenarm, or Donaghadee, &c., it will be seen that the four numbers at the same station have values alternately greater and less. This is owing, I conceive, to parallax, or some other cause which is periodical in one revolution of the moon nearly; and a correction is probably necessary, applicable with opposite signs to the alternate values. Thus, comparing the second with the mean of the first and third, half the difference is one value of the correction; comparing the third with the mean of the second and fourth, half the difference is another value of the correction; and the mean of these may be used. Thus corrected values of the “day of evanescence at low water — day of evanescence at high water” were obtained. Taking the means of the corresponding corrected values for the six stations from Glenarm to Dunmore East, we have,

About July	. 4 ^d ·41,	the mean value is	0 ^d ·76,	{ and the excess of the moon's } ^h ^m	20 46
				R.A. over the sun's R.A. is	
About July	. 17·86,	the mean value is	0·73	„ „ „	8 33
About August	1·92,	the mean value is	0·47	„ „ „	19 58
About August	14·56,	the mean value is	0·00	„ „ „	7 13

From the regularity of the progress of the numbers in the second and third columns, it appears certain that the value $7^h 13^m$ for α must be very near the truth.

From the reasoning above it will appear that, in the case of simultaneous evanescence of diurnal tide at high water and at low water, we have no means whatever of ascertaining the values of μ and ϕ on that day. Or, if we take the expressions on page 22, we have for diurnal tide at high water,

$$a.\sin\beta.\sin\{\alpha - \odot + A\} + b.\sin\{\alpha - \odot + B\},$$

and at low water,

$$a.\cos\beta.\sin\{\alpha - \odot + A\} + b.\cos\{\alpha - \odot + B\};$$

and, on the same day when both vanish,

$$\beta \text{ must} = \alpha' - \odot' + B \text{ and } a \sin\{\alpha' - \odot' + A\} \text{ must} = -b;$$

by which the expression for diurnal tide at high water is changed to

$$\begin{aligned} & a \cdot \sin\{\alpha' - \odot' + B\} \cdot \sin\{\alpha - \odot + A\} - a \cdot \sin\{\alpha' - \odot' + A\} \cdot \sin\{\alpha - \odot + B\} \\ & = a \cdot \sin(B - A) \cdot \sin\{\overline{\alpha - \odot} - \overline{\alpha' - \odot'}\}, \end{aligned}$$

and that at low water is changed to

$$\begin{aligned} & a \cdot \cos\{\alpha' - \odot' + B\} \cdot \sin\{\alpha - \odot + A\} - a \cdot \sin\{\alpha' - \odot' + A\} \cdot \cos\{\alpha - \odot + B\} \\ & = a \cdot \cos(B - A) \cdot \sin\{\overline{\alpha - \odot} - \overline{\alpha' - \odot'}\}. \end{aligned}$$

The maximum of the compound diurnal tide at high water will then be $a \cdot \sin(B - A)$, and that at low water $a \cdot \cos(B - A)$. From the observed values of these maxima we may obtain the values of a and $B - A$; but we cannot from these obtain either A or B , or $a \cdot \sin\beta$, or b ; and thus the separation of the solar and lunar diurnal tides is in this case impossible.

As it seemed probable that the same value of α ($7^{\text{h}} 13^{\text{m}}$), which was inferred from these stations, would apply to the other stations also, I changed the equations for w, x, y, z , to the form corresponding to $\alpha = 7^{\text{h}} 13^{\text{m}}$. But, on attempting to solve them for Kilbaha, Kilrush, &c., I was entirely baffled. I could not in any instance approach to a solution.

As a last resource I resorted to the following method. Although the observations cannot be supposed competent to furnish more than four unknown quantities, yet they may be combined, and with sufficiently favourable coefficients, for the determination of the six quantities w, x, wy, wz, xy, xz : as the equations will be simple, values can certainly be found for these quantities; and then the accuracy of the results will be tested by ascertaining whether the following equations hold; $\frac{wy}{w} = \frac{xy}{x}$,

$\frac{wz}{w} = \frac{xz}{x}$. In this manner then equations were formed and solved for the principal stations; but at none of them was there any approach towards satisfying the equations of condition which I have just given; nor even upon reducing the results to a common phase of the diurnal tide-wave, by means of the intervals of diurnal establishment to be given in Section IX., and taking the mean, was there any approach towards satisfying those equations. This is the last attempt that I made, and I confess myself, on the whole, completely unsuccessful.

The following considerations may perhaps explain the failure of all these attempts. We have seen that when the days of evanescence, of diurnal tide in semidiurnal high water, and of diurnal tide in semidiurnal low water, coincide, it is absolutely impossible to extract from the equations a result relating to the distinction of solar and lunar effects. It may therefore be inferred that, when the days of evanescence *nearly* coincide, the determination of the quantities sought will be *nearly* impossible, or will be liable to very great errors. Now, at all the stations the interval between

the days of evanescence for semidiurnal high water and for semidiurnal low water is small, and therefore it appears that in the case of the Irish tides in question, the difficulty of separating the solar and lunar effects is inevitable.

A treatment of the phenomena of diurnal tide which does not lead to a distinction of the solar and lunar effects must be considered as imperfect. And it has been explained that, though from a short series of observations the distinction cannot be extracted, it can with certainty be obtained from a long series of observations. With a full avowal of the completeness of my own failure and with a statement at the same time of what science requires and what it may reasonably expect, I may be permitted to explain an expression which I have used in my tract on Tides and Waves, Art. 564, where I have said that a determination by Mr. WHEWELL is worth little. My intention was to express that the distinction of the solar and lunar effects was theoretically important, that it might be obtained from the observations there referred to, and that it was not obtained. In stating that Mr. WHEWELL'S determination was worth little, my expression was thus far incorrect, that a general rule for the order of diurnal tides, though liable to some inaccuracy, was really obtained by Mr. WHEWELL. I regret that I should have made a statement which could thus seem to be insufficiently founded, and still more that I should have expressed it in a phrase which could be interpreted as lightly esteeming the deductions of the writer to whom we are indebted more than to any other for the knowledge which we possess regarding the laws of the tides.

Section V.—*Discussion of the height of apparent mean water, as deduced from the heights of high and low water only, corrected for diurnal tide; with reference to difference of station, and to variations of the magnitude of the tide and of the moon's declination.*

The heights at high and low water were corrected for the diurnal tide in height found in Section III. The age of the semidiurnal tide as deduced from heights having been found (by the process of the next section) to be about two days, the times were taken from the Nautical Almanac, at which the moon's hour-angle from the sun was 3^h, 9^h, 15^h, 21^h, and two days being added to these, the times were determined which were to be used as separating the large tides from the small tides. The groups of observations thus marked off were the following:—

1st Group, June	^d 28	^h 6 to July	^d 6	^h 16, small tide.
2nd Group, July	6	16 to July	13	3, large tide.
3rd Group, July	13	3 to July	20	9, small tide.
4th Group, July	20	9 to July	28	9, large tide.
5th Group, July	28	9 to August	5	0, small tide.
6th Group, August	5	0 to August	11	16, large tide.
7th Group, August	11	16 to August	18	15, small tide.

For each of these groups the mean of the heights at high water was taken, and the mean of the heights at low water was taken. Then the mean of the determinations of each of these classes in the 1st, 3rd, 5th, and 7th groups was taken, and thus was obtained a mean height at high water and a mean height at low water in small tides. The mean of these two means gave the Apparent Mean Height in small tides (so called in order to distinguish it from the Mean Height which will be found in Section XI.). By a similar treatment of the 2nd, 4th, and 6th groups, the mean height at high water, mean height at low water, and Apparent Mean Height, in large tides, were found. The results are as follows:—

Station.	Small Tides.			Large Tides.			Mean of Apparent Mean Heights.	Excess of Apparent Mean Height in large tides above Apparent Mean Height in small tides.
	Mean height at high water.	Mean height at low water.	Apparent Mean Height.	Mean height at high water.	Mean height at low water.	Apparent Mean Height.		
	ft.	ft.	ft.	ft.	ft.	ft.	ft.	ft.
Kilbaha	19·34	11·92	15·63	21·79	9·57	15·68	15·65	+ 0·05
Kilrush	20·50	12·06	16·28	22·96	9·77	16·37	16·32	+ 0·09
Foynes Island....	21·82	11·68	16·75	24·39	9·13	16·76	16·75	+ 0·01
Limerick.....	23·22	9·99	16·61	26·38	7·40	16·89	16·75	+ 0·28
Casleh Bay.....	21·81	14·12	17·97	24·43	11·52	17·98	17·97	+ 0·01
Galway	21·54	13·42	17·48	24·34	10·70	17·52	17·50	+ 0·04
Old Head	21·18	14·28	17·73	23·60	12·04	17·82	17·77	+ 0·09
Mullaghmore	21·22	14·78	18·00	23·42	12·92	18·17	18·08	+ 0·17
Buncrana	20·68	13·90	17·29	23·23	11·47	17·35	17·32	+ 0·06
Port Rush	18·95	16·23	17·59	20·35	15·13	17·74	17·66	+ 0·15
Carrowkeel	20·00	15·87	17·94	21·66	14·72	18·19	18·06	+ 0·25
Ballycastle	18·39	16·48	17·44	19·17	16·21	17·69	17·56	+ 0·25
Glenarm.....	20·08	15·11	17·60	20·60	14·67	17·64	17·63	+ 0·04
Donaghadee	21·73	13·83	17·78	23·12	12·56	17·84	17·81	+ 0·06
Ardglass.....	22·54	12·64	17·59	24·64	10·68	17·66	17·62	+ 0·07
Clogher Head....	21·85	12·30	17·08	23·91	10·28	17·10	17·09	+ 0·02
Kingstown	21·04	13·69	17·37	22·59	12·05	17·32	17·34	— 0·05
Dunmore East ..	19·72	11·82	15·77	21·57	10·08	15·83	15·80	+ 0·06
New Ross	21·23	11·92	16·58	23·31	10·42	16·87	16·72	+ 0·29
Passage West....	20·07	11·67	15·87	22·01	10·15	16·08	15·97	+ 0·21
Castle Townsend..	18·90	12·22	15·56	20·44	10·87	15·66	15·61	+ 0·10

The column which first demands our attention is the “Mean of Apparent Mean Heights.” The Apparent Mean Height increases in ascending the Shannon from Kilbaha to Limerick, and in ascending the Barrow from Dunmore East to New Ross. There is no difficulty in perceiving that such an effect must result from the mixture of a river-current with a tide, when the motion of the water is impeded by friction. But the Apparent Mean Height increases also from Kilbaha and Castle Townsend in going northward along both the eastern and the western coasts. And this occurs both at large tides and at small tides, with comparatively very little difference of amount. This conclusion will be fully supported by the results deduced by a more elaborate process from the whole mass of observations in Section XI. The details of levelling in the first section will scarcely permit us to ascribe any large part of this seeming difference of Apparent Mean Height to error in the instrumental process.

I regard this as a most important result, and I cannot with confidence offer any explanation of it.

The last column seems to show that the Apparent Mean Height is greater for large ranges of tide than for small ones, or that in spring tides the increase of elevation of the high water is greater than the increase of depression of the low water. A similar result has been found in the Thames. It does not appear to admit of explanation with our present theoretical knowledge. But in a period of two months it is impossible to separate the effects of varying range from those of varying declination of the moon and other varying circumstances.

For investigation of the effect of the moon's declination on the apparent mean height, the following process was used. It is to be remarked that (setting aside the differences of the moon's distance) the effect of southern declination is the same as that of northern declination; and the effect is proportional nearly to the square of declination; and therefore the groups into which our observations are to be divided are to be classified by large declinations (of either kind) and by small declinations; the separating time being nearly that at which the moon's declination $= \frac{1}{\sqrt{2}} \times$ moon's maximum declination. The times being thus found, one day was added to each, and thus the following separation of groups was made.

1st Group, June	^d 26	^h 3 to July	^d 3	^h 14, small declination.
2nd Group, July	3	14 to July	10	2, large declination.
3rd Group, July	10	2 to July	16	8, small declination.
4th Group, July	16	8 to July	23	10, large declination.
5th Group, July	23	10 to July	30	22, small declination.
6th Group, July	30	22 to August	6	12, large declination.
7th Group, August	6	12 to August	12	15, small declination.
8th Group, August	12	15 to August	19	16, large declination.

For each of these groups, the mean of the heights at high water was taken, and the mean of the heights at low water; and the mean of these gave a mean Apparent Mean Height in each group. The mean of the results from the 1st, 3rd, 5th and 7th groups, was used for small declinations, and the mean of the results from the 2nd, 4th, 6th and 8th, was used for large declinations. At Ballycastle the first group was wanting. Thus the following Table was found.

Station.	Apparent Mean Height with small declination.	Apparent Mean Height with large declination.	Excess of Apparent Mean Height with large declination above Apparent Mean Height with small declination.
	ft.	ft.	ft.
Kilbaha	15·64	15·66	+ 0·02
Kilrush	16·29	16·35	+ 0·06
Foynes Island	16·71	16·81	+ 0·10
Limerick	16·70	16·79	+ 0·09
Casleh Bay	17·94	18·00	+ 0·06
Galway	17·46	17·53	+ 0·07
Old Head	17·75	17·79	+ 0·04
Mullaghmore	18·05	18·13	+ 0·08
Buncrana	17·31	17·38	+ 0·07
Port Rush	17·64	17·70	+ 0·06
Carrowkeel	17·98	18·11	+ 0·13
Ballycastle	17·51	17·60	+ 0·09
Glenarm	17·56	17·69	+ 0·13
Donaghadee	17·77	17·86	+ 0·09
Ardglass	17·59	17·70	+ 0·11
Clogher Head	17·00	17·18	+ 0·18
Kingstown	17·26	17·45	+ 0·19
Dunmore East	15·77	15·82	+ 0·05
New Ross	16·72	16·69	− 0·03
Passage West	15·99	15·93	− 0·06
Castle Townsend	15·60	15·59	− 0·01

In like manner, a series of groups was formed, determined by the increase or decrease of the moon's declination without regard to its sign; one day being added to each of the times when the moon's declination was 0, and when it was maximum either north or south. Thus the following separation of groups was made.

1st Group, June	^d 22	^h 11	to June	^d 29	^h 21,	declination decreasing.
2nd Group, June	29	21	to July	7	0,	declination increasing.
3rd Group, July	7	0	to July	13	3,	declination decreasing.
4th Group, July	13	3	to July	19	19,	declination increasing.
5th Group, July	19	19	to July	27	4,	declination decreasing.
6th Group, July	27	4	to August	3	9,	declination increasing.
7th Group, August	3	9	to August	9	12,	declination decreasing.
8th Group, August	9	12	to August	16	0,	declination increasing.
9th Group, August	16	0	to August	23	11,	declination decreasing.

The mean of results from the 1st, 3rd, 5th, 7th and 9th groups was used for decreasing declination, and that of results from the 2nd, 4th, 6th and 8th, for increasing declination. At the first six stations, and at Ballycastle and Passage West, the first group was deficient. Thus the following Table was formed.

Station.	Apparent Mean Height with decreasing declination.	Apparent Mean Height with increasing declination.	Excess of Apparent Mean Height with decreasing declination over Apparent Mean Height with increasing declination.
	ft.	ft.	ft.
Kilbaha	15.65	15.64	+0.01
Kilrush	16.37	16.27	+0.10
Foynes Island .	16.76	16.74	+0.02
Limerick.	16.84	16.64	+0.20
Casleh Bay. . . .	17.96	17.95	+0.01
Galway	17.50	17.47	+0.03
Old Head	17.85	17.37	+0.48
Mullaghmore ..	18.23	17.99	+0.24
Buncrana	17.47	17.27	+0.20
Port Rush	17.82	17.57	+0.25
Carrowkeel. . . .	18.19	17.92	+0.27
Ballycastle	17.68	17.45	+0.23
Glenarm	17.73	17.59	+0.14
Donaghadee ..	17.91	17.77	+0.14
Ardglass	17.72	17.59	+0.13
Clogher Head..	17.17	17.05	+0.12
Kingstown	17.41	17.35	+0.06
Dunmore East .	15.87	15.76	+0.11
New Ross	16.85	16.61	+0.24
Passage West ..	16.05	15.89	+0.16
Castle Townsend	15.65	15.57	+0.08

It would seem from this that, if the apparent mean height depends upon the moon's declination, it is greatest when the moon's declination has decreased nearly to the point where the argument of declination is 135° or 315° (which makes the declination $= \frac{1}{\sqrt{2}} \times$ maximum declination). Thus at the middle of the observations, it is greatest about July 22. Now on July 22, the magnitude of the tide, though above the mean, was not maximum. From this circumstance, and from the decided superiority in magnitude of the excess found by classifying the tides by the moon's declination, over the excess found by classifying the tides by the magnitude of the tide, it appears extremely probable that the excess does really depend on the moon's declination. In that case, the greatest apparent mean height will occur about four days after the day of greatest declination.

Section VI.—*Discussion of the range of tide; and of semimenstrual inequality in height, apparent proportion of solar and lunar effects as shown by heights, and age of tide as shown by heights; from high waters and from low waters.*

The numbers from which we shall extract the results of this section are contained in the table of page 30. The means of the heights are made subservient to the accurate determinations of specific heights in the following manner. If θ is the difference of right ascension of the sun and moon, S and M their respective single effects, then (neglecting declinations, &c.) the actual height may be represented by

$A + M \sqrt{1 + \frac{2S}{M} \cos 2\theta + \frac{S^2}{M^2}}$. If we expand this and integrate from $\theta = -45^\circ$ to

$\theta = +45^\circ$, and divide by $\frac{\pi}{2}$, we obtain the expression for the mean of large tides. If we integrate from $\theta = 45^\circ$ to $\theta = 135^\circ$, and divide by $\frac{\pi}{2}$, we obtain the expression for the mean of small tides. The difference between these, which will be found to be $\frac{4}{\pi}S - \frac{2}{3\pi} \cdot \frac{S^3}{M^2}$, is the same as the difference between the second and fifth columns in the table of p. 30, if high water is under consideration, or as the difference between the third and sixth columns, if low water is under consideration, or as the difference between the 2nd column - 3rd, and 5th column - 6th, if the range is considered. The second term of the formula may sometimes be omitted: and then we have $\frac{4}{\pi}S = \text{difference}$, and $S = \frac{\pi}{4} \times \text{difference}$, S being the solar effect in the elevation or range under consideration. If we add this to the mean elevation, which is represented by $A + M$ nearly, we shall have $A + M + S$ which is the absolute maximum; and if we subtract it, we have $A + M - S$, which is the absolute minimum. The same applies to range, with this difference only, that the constant A will be eliminated in subtracting the heights at low water from those at high water.

In order to obtain the value of $\frac{S}{M}$, we may remark that the difference of the two means divided by half the sum is $\frac{\frac{4}{\pi}S - \frac{2}{3} \frac{S^3}{M^2}}{\pi \left(1 + \frac{1}{4} \frac{S^2}{M^2}\right)}$; and if we call this δ , we easily find

$$\frac{S}{M} = \frac{\pi}{4} \delta \left\{ 1 + \frac{5}{12} \left(\frac{\pi}{4} \delta \right)^2 \right\}, \text{ in which the last term is small.}$$

Neither of these expressions is perfectly correct, because they assume that the tidal day is always of the same length.

By means of these formulæ, and the numbers on page 30, the following Table is formed.

Station.	Difference of mean heights in large tides and in small tides.		Proportion of difference at high water to difference at low water.	Range in mean of large tides.	Range in mean of small tides.	Difference.	Mean range, or M.	Difference divided by mean range.	Corresponding value of $\frac{S}{M}$.	Absolute maximum range.	Absolute minimum range.	Half difference, or S.
	High water.	Low water.										
	ft.	ft.	ft.	ft.	ft.	ft.	ft.			ft.	ft.	ft.
Kilbaha	2.45	2.35	1.04	12.22	7.42	4.80	9.82	0.49	0.41	13.8	5.8	4.0
Kilrush	2.46	2.29	1.07	13.19	8.44	4.75	10.82	0.44	0.36	14.7	6.9	3.9
Foynes Island .	2.57	2.55	1.01	15.26	10.14	5.12	12.70	0.40	0.33	16.9	8.5	4.2
Limerick.	3.16	2.59	1.22	18.98	13.23	5.75	16.11	0.36	0.29	20.8	11.4	4.7
Casleh Bay . . .	2.62	2.60	1.01	12.91	7.69	5.22	10.30	0.51	0.43	14.7	5.9	4.4
Galway	2.80	2.72	1.03	13.64	8.12	5.52	10.88	0.51	0.43	15.6	6.1	4.7
Old Head	2.42	2.24	1.08	11.56	6.90	4.66	9.23	0.50	0.42	13.1	5.3	3.9
Mullaghmore ..	2.20	1.86	1.18	10.50	6.44	4.06	8.47	0.48	0.40	11.9	5.1	3.4
Buncrana	2.55	2.43	1.05	11.76	6.78	4.98	9.27	0.54	0.46	13.6	5.0	4.3
Port Rush	1.40	1.10	1.27	5.22	2.72	2.50	3.97	0.63	0.55	6.0	1.8	2.2
Carrowkeel. . . .	1.66	1.15	1.44	6.94	4.13	2.81	5.54	0.51	0.43	7.9	3.1	2.4
Ballycastle . . .	0.78	0.27	2.90	2.96	1.91	1.05	2.44	0.43	0.36	3.3	1.5	0.9
Glenarm	0.52	0.44	1.18	5.93	4.97	0.96	5.45	0.18	0.14	6.2	4.7	0.8
Donaghadee ..	1.39	1.27	1.10	10.56	7.90	2.66	9.23	0.29	0.23	11.3	7.1	2.1
Ardglass	2.10	1.96	1.07	13.96	9.90	4.06	11.93	0.34	0.27	15.1	8.7	3.2
Clogher Head. .	2.06	2.02	1.02	13.63	9.55	4.08	11.59	0.35	0.28	14.8	8.4	3.2
Kingstown	1.55	1.64	0.95	10.54	7.35	3.19	8.95	0.36	0.29	11.6	6.4	2.6
Dunmore East .	1.85	1.74	1.06	11.49	7.90	3.59	9.70	0.37	0.30	12.6	6.8	2.9
New Ross	2.08	1.50	1.39	12.89	9.31	3.58	11.10	0.32	0.26	14.0	8.2	2.9
Passage West . .	1.94	1.52	1.28	11.86	8.40	3.46	10.13	0.34	0.27	12.8	7.4	2.7
Castle Townsend	1.54	1.35	1.14	9.57	6.68	2.89	8.13	0.36	0.29	10.5	5.7	2.4

The numbers in this table present several subjects for our consideration.

First, it is to be remarked that the difference between the numbers of the second and third columns corresponds to the number in the last column of the table on page 30 (it is, in fact, the double of that number); but the exhibition of the numbers in this form serves, in some instances, to point out their origin more distinctly. At Limerick, for instance, it appears that a large tide produces less effect on the low water than on the high water; and the reason evidently is, that the height of low water there depends more upon the freshwater current of the river Shannon, than upon the state of the water in the sea, whereas the height of high water depends mainly on the sea. The same explanation applies at New Ross and at Passage West. It does not however apply to Mullaghmore, Port Rush, Ballycastle, or Glenarm; nor, with perfect certainty, to Carrowkeel. I must refer to Section X. and some of the succeeding sections, for a statement of the laws of the individual tides at these places; and shall only remark here, that much remains to be done with the theoretical treatment of the motion of waves, before the tides on the north-eastern coast can be fully explained.

The next column which deserves particular attention is that for $\frac{S}{M}$. From the agreement among the numbers at the littoral stations on the western coast (Kilbaha, Casleh Bay, Galway, Old Head, Mullaghmore), it seems certain that the value of $\frac{S}{M}$, in this part of the Atlantic, is pretty exactly 0.42. At Brest, in a position equally

exposed to the open sea, LAPLACE found $\frac{S}{M} = 0.33$. I imagine that, as regards the sun's position in declination, the present series of tides may be supposed to give a value for $\frac{S}{M}$ differing very little from the mean, but rather too small than too great. If so, here is an undoubted discordance. Yet it is remarkable that the series of numbers in this table presents a still greater discordance under circumstances where we should hardly expect it. The littoral station of Castle Townsend is fully open to the Atlantic; and those of Dunmore East, Kingstown, and Clogher Head, are more and more subject to the effects (whatever they may be) of the inclosure of the Irish Channel. Yet all these stations, including Castle Townsend, give for $\frac{S}{M}$ a value of about 0.28.

In speculating on the causes of these discordances, it is important to observe, that the relation of the successive changes of a progressive tide may be considered in two different ways. One of these ways is applicable properly to rivers and similar channels; it consists in assuming that each semidiurnal tide is nearly independent of every other semidiurnal tide, and that the quantities of the second and higher orders of the range are sensible*. The other is applicable to open seas, where the vertical oscillation is insignificant in comparison with the depth, and where the alteration of the horizontal surface, by the shoaling of the shore, is perfectly unimportant; here the principle of the superposition of small movements applies, and the solar and lunar tides may be considered as perfectly independent. Thus considered, the discordance to which I have alluded may be stated thus: the lunar tide at Castle Townsend is less than that at Kilbaha, in the proportion of 8.13 to 9.82, but the solar tide at Castle Townsend is less than that at Kilbaha, in the proportion of 2.4 to 4.0. There can be no doubt that in the lunar tide, the difference between 8.13 and 9.82 depends (among other circumstances) upon the periodic time of the tide, inasmuch as a tide infinitely slow would produce the same effect at both stations; and therefore we must not be surprised that the solar tide, whose recurrence is more rapid than that of the lunar tide, should undergo a larger proportional alteration, like that between 2.4 and 4.0.

In another section I shall show that between Dunmore East and Kingstown, the tide is suddenly and completely reversed, high water at one of these stations corresponding exactly (in time) to low water at the other; while from Dunmore East to Donaghadee, there is scarcely any perceptible difference of phase. It is worthy of remark, that during these changes and coincidences, the value of $\frac{S}{M}$ is unaltered.

The remarkable change in the values of $\frac{S}{M}$ at Port Rush, Ballycastle, and Glenarm, depends also, without doubt, upon the difference of the modifications produced in the

* Encyclopædia Metropolitana, *Tides and Waves*, Section IV., Subsection 3.

solar and in the lunar tides, by the narrow channel which, beginning with Islay (opposite Port Rush), continues to the Mull of Galloway (opposite Donaghadee). But I profess myself totally unable to explain with greater accuracy why, in passing from Mullaghmore to Ballycastle, the principal reduction of lunar tide takes place between Mullaghmore and Port Rush, while the principal reduction of solar tide takes place between Port Rush and Ballycastle.

The successive diminution in the values of $\frac{S}{M}$ from Kilbaha to Kilrush, Foynes Island, and Limerick, as well as the reduction from Dunmore East to New Ross, and that from Port Rush to Buncrana and Carrowkeel, are to be explained from the other consideration which I have described as applicable to rivers. In fact, in these cases, from the very greatly contracted current-way at low water, the effect of variation in the depression of the ocean low water is very small, and therefore the effect of semi-menstrual inequality is sensible almost only in the high water, and its whole effect on the range is therefore less than it ought to be.

Before dismissing this table, I shall remark, that the very great diminution in the range of tide in the North Channel, makes it impossible for us to believe that the tide in the Irish Channel is supplied from the North Channel, however much the consideration of the times in the next section may lead us, in the first instance, to imagine so.

I will now proceed with the age of the tide as shown by the heights. The method employed was the following:—

The mean heights having been found from the preceding tables, the heights for every day (corrected for diurnal tide) were examined, and the time was ascertained as nearly as practicable at which the height coincided with the mean height. These times were then compared with the times at which the moon's hour-angle from the sun was 3^h, 9^h, 15^h, and 21^h; namely, June, 26^d 6^h; July, 4^d 16^h, 11^d 3^h, 18^d 9^h, 26^d 9^h; August, 3^d 0^h, 9^d 16^h, 16^d 15^h. High waters and low waters were treated separately. At each place eight comparisons were obtained for high water and eight for low water: excepting that in the high waters, no comparison was made at Old Head with July 11^d 3^h, and 18^d 9^h (the times of mean height being very uncertain); nor at Ballycastle, with July 18^d 9^h and 26^d 9^h (for the same reason), nor with June 26^d 6^h (the observations having commenced too late); nor at Glenarm, Donaghadee, or Kingstown, with July 18^d 9^h and 26^d 9^h (the times being uncertain). The results for each place being collected, and the means being taken, the following Table was formed:—

Age of Tide as inferred from Heights.

Station.	High water.		Low water.		Station.	High water.		Low water.	
	d	h	d	h		d	h	d	h
Kilbaha	2	2	2	0	Ballycastle	2	2	2	17
Kilrush	1	21	2	3	Glenarm	1	7	2	6
Foynes Island	2	2	2	4	Donaghadee	1	19	2	17
Limerick	2	2	2	7	Ardglass	2	5	2	11
Casleh Bay	2	0	2	2	Clogher Head	2	4	3	4
Galway	2	2	2	2	Kingstown	1	20	2	13
Old Head	1	15	2	6	Dunmore East	2	1	2	14
Mullaghmore	1	22	2	8	New Ross	2	2	2	14
Buncrana	2	1	2	5	Passage West	1	21	2	7
Port Rush	1	17	2	5	Castle Townsend	1	21	2	9
Carrowkeel	1	17	2	6					

In the Encyclopædia Metropolitana, *Tides and Waves*, Art. 545, I have given reasons for believing that the true age of the tide is that given by the heights. From the inspection of this table it appears certain that the age of the tide on the western coast of Ireland is almost exactly two days. The different results are upon the whole very consistent. Yet there are some discordances which I cannot entirely explain. The most distinctly marked discordance is this—that the age given by the Low Waters is, in every instance except one, greater than that given by the High Waters. I have shown, I think, with certainty, in the Encyclopædia Metropolitana, *Tides and Waves*, Articles 452 and 544, that the cause of the principal part of the apparent age of tide is friction. It appears to me not unlikely, that, on carrying out the theory of friction in combination with the consideration of oscillations bearing a finite proportion to the depth of the sea, the apparent age of the tide might be found greater for low waters than for high waters. But I have not examined the theory so far as to feel myself warranted in positively assigning this as the explanation.

Section VII.—*Establishment of each port, and progress of semidiurnal tide round the island.*

The process used in this investigation is very nearly similar to that by which the table in page 30 was formed. The mean interval from the moon's transit at Greenwich to the time of tide was found (by the operations of the next section) to coincide with the actual interval nearly at the following times:—Port Rush and Ballycastle (high and low), June, 22^d 9^h, 30^d 20^h; July, 7^d 19^h, 14^d 21^h, 21^d 23^h, 30^d 16^h; August, 6^d 4^h, 13^d 5^h, 20^d 16^h; Kilbaha, Kilrush, Casleh Bay, Galway, Mullaghmore, Buncrana, Carrowkeel, Glenarm, Donaghadee, Ardglass, Clogher Head, Kingstown, Castle Townsend, (high and low), 1^d 8^h later; Foynes Island, Old Head, Dunmore East, Passage West (high and low), Limerick (high only), 20^h later than the last; New Ross (high and low), 23^h later than the last; Limerick (low only), 22^h later than the last, or 4^d 1^h later than at Port Rush and Ballycastle. The intervals from the moon's transit to the time of tide being taken for every tide (the high water, 1st division, being referred to the upper transit, the low water, 1st division, to the transit over the

meridian 6^h west, the high water, 2nd division, to the lower transit, and the low water, 2nd division, to the transit over the meridian 18^h west), groups were formed divided by the times above mentioned, and the mean of the intervals was taken for each group. Thus the following Table was formed:—

Station.	High water.			Low water.			Mean of both.
	Mean of large intervals.	Mean of small intervals.	Mean.	Mean of large intervals.	Mean of small intervals.	Mean.	
Kilbaha	h m 5 11	h m 4 23	h m 4 47	h m 5 6	h m 4 12	h m 4 39	h m 4 43
Kilrush	5 22	4 31	4 56	5 20	4 28	4 54	4 55
Foynes Island.	6 15	5 30	5 52	6 0	5 12	5 36	5 44
Limerick.	6 46	6 0	6 23	7 14	6 17	6 45	6 34
Casleh Bay	5 21	4 30	4 55	5 11	4 14	4 42	4 48
Galway	5 28	4 40	5 4	5 11	4 13	4 42	4 53
Old Head	5 22	4 47	5 4	5 22	4 45	5 3	5 4
Mullaghmore.	6 2	5 9	5 35	5 53	5 7	5 30	5 33
Buncrana	6 45	5 39	6 12	6 34	5 36	6 5	6 8
Port Rush	7 29	6 0	6 45	7 14	5 59	6 36	6 40
Carrowkeel.	8 11	7 2	7 36	8 1	7 4	7 32	7 34
Ballycastle	8 37	6 14	7 25	8 47	7 3	7 55	7 40
Glenarm.	11 1	10 26	10 43	11 7	10 22	10 44	10 44
Donaghadee	11 21	10 42	11 1	11 23	10 36	11 0	11 1
Ardglass.	11 21	10 40	11 0	11 25	10 38	11 2	11 1
Clogher Head.	11 41	10 54	11 17	11 34	10 47	11 10	11 14
Kingstown	11 52	11 6	11 29	11 29	10 41	11 5	11 17
Dunmore East	5 32	4 47	5 9	5 41	4 56	5 18	5 13
New Ross	6 17	5 36	5 56	6 55	6 9	6 32	6 14
Passage West.	5 38	4 55	5 16	5 52	5 9	5 30	5 23
Castle Townsend.	5 16	4 29	4 52	5 19	4 30	4 54	4 53

On inspecting the intervals for Castle Townsend, Kilbaha, Casleh Bay, Galway, Old Head, and Mullaghmore, it appears evident that the semidiurnal tide approaches the western coast of Ireland very nearly from the west, or possibly from a direction a little south of the west. On examining the intervals at Port Rush, Ballycastle, Glenarm, Donaghadee, their regular progression seems at first to show that the tide wave enters the Irish Sea by way of the North Channel. But the objections to this supposition are most serious. Theoretically it is certain that (even without regarding the effects of friction) when a tide-wave passing through a narrow channel enters into a wide one its vertical range will be diminished*. But here the mean range in the narrow channel at Ballycastle is only 2½ feet and at Glenarm 5½ feet: this tide-wave therefore could not possibly produce a tide of more than one or two feet in the Irish Sea; whereas, from Ardglass southward to Kingstown, the range is from ten to twelve feet. It seems therefore that this supposition may at once be dismissed; and the only supposition which can be substituted for it is, that the tide-wave enters the Irish Sea by St. George's Channel. But here a most remarkable circumstance occurs. A reference to the table above will show that the high water at Kingstown occurs six hours and a few minutes later than that at Dunmore East, or, in other words, that

* Encyclopædia Metropolitana, *Tides and Waves*, Art. 264.

the high water at Kingstown coincides *precisely* with the low water at Dunmore East, and *vice versâ*. Moreover, between these two stations occurs the station Courtown; and here, as will appear in Section XVII., the semidiurnal tide is nearly insensible. The difference in the times at Dunmore East and Kingstown does not therefore arise from a slow transmission of tide; but arises from a sudden *inversion* of the wave, the point which separates elevation from depression being not far from Courtown. And the question now is, whether, on the supposition that the tide-wave enters the Irish Sea by this southern entrance, it is possible to explain the existence of this neutral point and the inversion of the tide beyond it.

The explanation which I have to give rests upon a proposition long known to me as a matter of theory, but for which I never expected to find a practical application. In the Encyclopædia Metropolitana, *Tides and Waves*, Art. 307, I have shown that when the tidal wave enters a gulf (considered as a canal of uniform breadth and depth, stopped by a transverse barrier), the expression for the elevation of the water at

the time t at a point whose distance from the sea is x , is $\frac{A}{\cos ma} \sin \overline{nt} + B \cdot \cos \overline{ma - mx}$;

where n is the constant proper to the periodic time of the tide-wave, $\frac{n}{m} = \sqrt{g \times \text{depth}}$,

and a is the whole length of the canal. This expression shows that the vertical oscillations are simultaneous throughout; the coefficient in each place being

$\frac{A}{\cos ma} \cos \overline{ma - mx}$. Now if the canal is so long and so shallow that ma is greater

than $\frac{\pi}{2}$; then, on taking $x = a - \frac{\pi}{2m}$ the coefficient vanishes; and on taking x greater

than $a - \frac{\pi}{2m}$ the coefficient changes its sign. This case then agrees, as regards the

simultaneity and the inversion, with the case of observation before us, and all that is necessary for its complete application is, that the virtual head of the channel be supposed to be at a distance from Courtown equal to that which a *progressive* wave

would pass over in $3^{\text{h}} 5^{\text{m}}$ (which would make $ma = \frac{\pi}{2}$). Without professing myself

able to enter into details upon the depth of the water and the form of the supposed channel, I express my belief that this solution does accurately apply here: and I regard it as one of the most remarkable cases that has ever been noticed in the observations of tides.

Assuming then that the tide of the Irish Sea is explained, we have only further to explain the apparent passage of the tide through the North Channel. This is merely a case of the proposition in Art. 310 of the *Tides and Waves*, relating to a canal joining two tidal seas, from which it appears that there will always be in such a canal an apparent passage of the tide-wave in the same direction at every part. To represent the circumstances completely, it would be necessary to introduce the consideration of friction.

The only remaining result of the table in page 39 which deserves attention here, is the difference of the times occupied by the rise of the water and by the fall of the water. In the river stations (Limerick and New Ross) the fall of the water occupies a longer time than the rise. This, as a consequence of theory, is explained in Art. 206 of the *Tides and Waves*. At most of the other stations, the rise appears to occupy a very little longer time than the fall. This, however, as it depends on the estimation of the times of high and low water, is subject to great doubt: the terms upon which such difference depends will be investigated with great accuracy in Sections X. and XVI.

Section VIII.—*Semimenstrual inequality in time, proportion of solar and lunar effects as shown by times, and apparent age of tide as shown by times; from high water and from low water.*

The interval from the moon's transit over the meridian to high water (and similarly from the moon's transit over the six-hour meridian to low water) is theoretically expressed by $E + F$, where E is a constant, and $\tan 2F = \frac{\frac{S}{M} \sin 2\theta}{1 + \frac{S}{M} \cos 2\theta}$, θ being the hour-

angle of the moon from the sun. The declinations, &c. are supposed here to have their mean values. Investigating from this the expression for F , integrating from $\theta=0$ to $\theta=\frac{\pi}{2}$, and dividing by $\frac{\pi}{2}$, we obtain the value of the mean of large intervals; performing the same operation from $\theta=\frac{\pi}{2}$ to $\theta=\pi$, we obtain the value of the mean of small intervals. The difference which will be found $= \frac{2}{\pi} \cdot \frac{S}{M} \cdot \left(1 + \frac{1}{9} \left(\frac{S}{M}\right)^2\right)$, is the difference between the 2nd and 3rd columns or between the 5th and 6th columns in page 39, expressed in arc; or, as 2π of arc in the estimation of θ and F correspond to a tidal day of 1488^m, if we put i for the number of minutes in that difference, the equation is

$$\frac{2}{\pi} \cdot \frac{S}{M} \cdot \left(1 + \frac{1}{9} \left(\frac{S}{M}\right)^2\right) = i \times \frac{2\pi}{1488}.$$

From this we obtain

$$\frac{S}{M} = \frac{\pi^2 \cdot i}{1488} \times \left\{ 1 - \frac{1}{9} \cdot \left(\frac{\pi^2 i}{1488}\right)^2 \right\} \text{ nearly;}$$

and then the maximum value of F in arc will be found by making $\sin 2F' = \frac{S}{M}$, and converting F' into time by the proportion stated above. Thus the following Table is formed.

Station.	High water.			Low water.			Mean of values of $\frac{S}{M}$.
	Difference of means of large intervals and small intervals.	Value of $\frac{S}{M}$.	Maximum semimenstrual inequality \pm .	Difference of means of large intervals and small intervals.	Value of $\frac{S}{M}$.	Maximum semimenstrual inequality \pm .	
	m		m	m			
Kilbaha	48	0·32	38·5	54	0·35	42	0·34
Kilrush	51	0·34	41	52	0·34	41	0·34
Foynes Island	45	0·30	36	48	0·32	38·5	0·31
Limerick	46	0·31	37	62	0·40	48·5	0·35
Casleh Bay	51	0·34	41	57	0·37	44·5	0·36
Galway	48	0·32	38·5	58	0·37	44·5	0·34
Old Head	35	0·23	27	37	0·24	28·5	0·24
Mullaghmore	53	0·35	42	46	0·31	37	0·33
Buncrana	66	0·42	51	58	0·37	44·5	0·39
Port Rush	89	0·57	71·5	75	0·49	60	0·53
Carrowkeel	69	0·45	54·5	57	0·37	44·5	0·41
Ballycastle	143	0·82	113	104	0·65	83	0·74
Glenarm	35	0·23	27	45	0·30	36	0·26
Donaghadee	39	0·25	30	47	0·31	37	0·28
Ardglass	41	0·27	32	47	0·31	37	0·29
Clogher Head	47	0·31	37	47	0·31	37	0·31
Kingstown	46	0·31	37	48	0·32	38·5	0·31
Dunmore East	45	0·30	36	47	0·31	37	0·31
New Ross	41	0·27	32	46	0·31	37	0·29
Passage West	43	0·28	33·5	43	0·28	33·5	0·28
Castle Townsend . . .	47	0·31	37	49	0·33	39·5	0·32

I have stated in the *Encyclopædia Metropolitana*, *Tides and Waves*, Art. 538, that I consider the values of $\frac{S}{M}$ deduced from the semimenstrual inequalities in time to be real and certain representations of the proportions of the sun's effect to the moon's effect in the seas in the neighbourhood of each station; those deduced from the heights being liable to the effects of many local disturbing causes which do not affect those deduced from the times. In this view the table above deserves consideration. The littoral stations (including those in the Irish Sea) agree in giving a value of 0·32 or 0·33, nearly the same as that found at Brest by LAPLACE and Sir J. W. LUBBOCK. But at Port Rush and Ballycastle (the first stations in the North Channel) the lunar tide appears to be diminished in a far greater proportion than the solar tide. And then, after this alteration of relative magnitude has been established in the open sea of the neighbourhood, it appears to be again nearly destroyed by some local cause affecting the heights, so that in the table of page 35, the value of $\frac{S}{M}$ is restored to its average value. As far as the observations can be trusted for accuracy, the two conclusions which I have mentioned appear at first sight perfectly certain; for the greatest difference of intervals from moon's transit (on which the value of $\frac{S}{M}$ in this page depends) is deduced from comparisons of the times of tides of equal vertical range, in which therefore the stream of tide in the neighbouring channels of small depth and width, &c. was the same, and therefore could not disturb the difference of times.

But the value in page 35 is deduced from the comparison of high and low tides, in which the stream of tides, &c. is different. The second apparent alteration can take place only where the tide has arrived at such localities that the second order of the vertical oscillation produces sensible terms. I was at first misled by the plausibility of this reasoning.

Its fallacy, or rather its error, will appear from the following considerations. The semimenstrual inequality in time which theoretically is proper for giving the value of $\frac{S}{M}$ is that which depends only upon those differences of time which are caused by difference in the relative positions of the sun and moon, when the magnitudes of the tides are exactly the same. But when there also exists a difference of time caused by the difference of magnitude of the tide (having its maximum nearly at the time of evanescence of the proper semimenstrual inequality), then these two differences or inequalities are combined, forming a single inequality whose time of evanescence is different from those of both the original inequalities, and whose magnitude is greater than the magnitude of either. Thus it appears that the gross semimenstrual inequality in time must not be used for estimation of $\frac{S}{M}$. A correct application of these principles, and a consequent harmony of results, will be seen in Section XIV.

At every station except Mullaghmore, the value of $\frac{S}{M}$ in the table above appears greater at low water than at high water. This evidently depends upon the difference in times (as affected by magnitude of tide) for low water and for high water, which is combined as above stated with the proper semimenstrual inequality.

MR. WHEWELL, in his invaluable memoirs on cotidal lines, stated that there were great contradictions in the accounts of the establishment of Ballycastle. The numbers above serve in some degree to explain this. The semimenstrual inequality alone, comparing observations taken when its value was maximum positive with those taken when its value was maximum negative, would produce nearly four hours of uncertainty. More than half an hour (see the table in page 19) might be added by the diurnal tide.

I shall now proceed with the age of tide as shown by the times. The method employed was the same as for the heights, in forming the table in page 38. The times were ascertained at which the interval from moon's transit over the meridian to high water (and similarly the interval from moon's transit over the 6-hour meridian to low water), corrected for diurnal tide, agreed with the mean interval for high water (or for low water) in page 39. These times were then compared with the times at which the moon's hour-angle from the sun was 0^h , 6^h , 12^h , 18^h ; namely, June, $22^d 9^h$, $30^d 20^h$; July, $7^d 19^h$, $14^d 21^h$, $21^d 23^h$, $30^d 16^h$; August, $6^d 4^h$, $13^d 5^h$, $20^d 16^h$. The high and low waters were treated separately. At Ballycastle low water six results only were obtained; for all the other determinations seven or eight results were compared. Thus the following Table was formed.

Age of Tide as inferred from Times.

Station.	High water.	Low water.	Station.	High water.	Low water.
	d h	d h		d h	d h
Kilbaha	1 8	1 5	Ballycastle	-0 11	-0 19
Kilrush	1 13	1 10	Glenarm	+1 14	1 9
Foynes Island.	1 21	2 2	Donaghadee	1 7	1 9
Limerick.	1 22	4 1	Ardglass.	1 8	1 15
Casleh Bay.	1 14	0 22	Clogher Head.	1 10	1 12
Galway	1 10	1 7	Kingstown	0 22	0 23
Old Head	2 10	2 9	Dunmore East	2 11	2 8
Mullaghmore	1 6	1 11	New Ross	2 20	3 11
Buncrana	1 4	1 3	Passage West.	2 2	1 19
Port Rush	0 4	0 7	Castle Townsend..	1 12	1 6
Carrowkeel.	1 0	1 1			

In the *Tides and Waves*, Art. 463 and 465, I have shown that the age of tide inferred from the times of high water in a river (where spring-tide high waters pass more rapidly than neap-tide high waters) is too small, and that the age of tide inferred from the times of low water in a river (where spring-tide low waters pass more slowly than neap-tide low waters) is too great. These propositions, at least the second, are well illustrated at Limerick and New Ross. For the other stations I feel myself in some difficulty. With the exception only of Old Head, Dunmore East, and Passage West, all the ages of tide above are too small, for low water as well as for high water. This requires us to assume that all the phases of the tide-wave (low water as well as high water) are transmitted over the sea more rapidly in the spring-tides than in the neap-tides. I conjecture that some theory of friction may possibly explain this. It cannot be explained by supposing the second power of the small movements sensible; for on that assumption the age of tide given by low water would be increased.

It is worthy of remark that at Ballycastle the effects depending on the position of the sun and moon *appear* to precede their cause.

Section IX.—*Formation of the time of diurnal high water; progress of the diurnal tide-wave round the island; comparison of its progress and range with those of the semidiurnal tide.*

In page 20 I have given a table of the maximum values of diurnal tide, at high water and at low water. The diurnal tide being supposed to follow the law of sines, its maximum coefficient will be found by taking the square root of the sum of the squares of those two values, and the time after semidiurnal high water at which diurnal high water occurs will be found by taking the angle whose tangent = $\frac{\text{value at low water}}{\text{value at high water}}$, and converting that angle into time at the rate of 360° for a lunar day. The maximum diurnal tide for semidiurnal high water and that for semidiurnal low water may be conceived to hold for any day near to the day of absolute maximum. Thus the following Table is formed.

Station.	July 9.		July 23.		August 6.	
	Coefficient of diurnal tide.	Interval from semidiurnal high water, first division, to diurnal high water.	Coefficient of diurnal tide.	Interval from semidiurnal high water, first division, to diurnal high water.	Coefficient of diurnal tide.	Interval from semidiurnal high water, first division, to diurnal high water.
	ft.	h m	ft.	h m	ft.	h m
Kilbaha	0.55	20 44	0.71	9 5	0.58	20 36
Kilrush	0.56	20 4	0.57	9 0	0.53	20 25
Foynes Island . .	0.81	20 9	0.69	8 14	0.67	20 31
Limerick	1.01	19 50	0.69	9 11	0.64	20 50
Casleh Bay	0.69	20 12	0.66	8 44	0.63	20 28
Galway	0.81	19 49	0.71	8 37	0.43	20 55
Old Head	0.83	20 56	0.85	9 18	0.98	21 18
Mullaghmore . . .	0.89	23 25	0.82	11 34	0.86	24 18
Buncrana	0.64	0 19	0.76	13 4	0.70	1 26
Port Rush	0.52	1 5	0.63	14 2	0.69	2 4
Carrowkeel	0.47	1 28	0.95	13 1	0.68	1 58
Ballycastle	0.50	0 24	0.68	13 11	0.60	1 4
Glenarm	0.96	22 8	0.79	10 3	0.99	22 19
Donaghadee	0.91	21 44	0.85	10 16	1.03	21 40
Ardglass	0.82	21 34	0.84	10 23	0.91	21 55
Clogher Head . . .	0.85	22 2	0.79	10 21	0.83	21 42
Kingstown	0.76	21 38	0.67	10 3	0.83	21 58
Dunmore East . . .	0.21	2 37	0.33	15 10	0.27	1 38
New Ross	0.22	0 51	0.40	14 33	0.25	0 47
Passage West	0.15	4 2	0.22	14 13	0.17	1 20
Castle Townsend . .	0.16	18 3	0.09	12 41	0.10	20 52

A glance at this table will show how different are the velocities of the diurnal tide-wave and the semidiurnal tide-wave. From Kilbaha to Port Rush, the diurnal tide travels in a direction so different, or with a velocity so small, that it loses $5\frac{1}{2}$ hours in time upon the semidiurnal wave. But it passes through the North Channel with such speed that at Donaghadee it has regained about $4\frac{1}{2}$ hours. Its course however will be better understood by forming the actual time of high diurnal tide, or rather its interval after the moon's transit. I have treated the numbers in the following manner:—Increasing the numbers for July 23rd by $12^h 24^m$, or half a tidal day (because the moon's declination then was in a direction opposite to that on July 9th and August 6th), I have three comparable intervals from semidiurnal high water to diurnal high water. I take the mean of these, and apply it to the mean interval from moon's transit to semidiurnal high water in the table of page 39. I also take the mean of the three coefficients. Thus the following Table is formed; in which it is to be remembered that the coefficients are to be taken positive for July 9th and August 6th, and negative for July 23rd.

Station.	Coefficient of diurnal tide.	Interval from moon's transit to diurnal high water.	Station.	Coefficient of diurnal tide.	Interval from moon's transit to diurnal high water.
	ft.	h m		ft.	h m
Kilbaha	0·61	0 51	Ballycastle	0·59	8 25
Kilrush	0·55	0 45	Glenarm	0·91	8 15
Foynes Island	0·72	1 22	Donaghadee	0·93	8 14
Limerick	0·78	2 31	Ardglass	0·86	8 18
Casleh Bay	0·66	0 38	Clogher Head	0·82	8 36
Galway	0·65	0 40	Kingstown	0·75	8 30
Old Head	0·89	1 35	Dunmore East	0·27	7 33
Mullaghmore	0·86	4 39	New Ross	0·29	7 30
Buncrana	0·70	6 56	Passage West	0·18	7 47
Port Rush	0·61	8 16	Castle Townsend	0·12	1 25
Carrowkeel	0·70	8 55			

Comparing the coefficients in this table with the column of mean range of semi-diurnal tide in page 35, we can discover no analogy between them. The range of diurnal tide is not at all reduced in the North Channel, where the semidiurnal tide is so much diminished; nor (as will also be shown hereafter) is it particularly diminished between Kingstown and Dunmore East, where the semidiurnal tide is nearly or quite obliterated; but it is much diminished at Castle Townsend, where the semi-diurnal tide is pretty large.

On examining the interval from moon's transit, it appears evident that the diurnal tide comes from the south, or very nearly from the south. It appears also that it does not pass in either direction through the North Channel, but that the strait is filled simultaneously, or nearly so, at both ends. It appears also that the wave travels very quickly from south to north in the Irish Channel; so quickly indeed that it is probable that the tide is simultaneous throughout. But between Castle Townsend and Passage West it loses more than six hours, or a quarter of a diurnal tide. I am totally unable to explain this. The case is very greatly different from that discussed in page 40, where the change of phase was almost exactly half a tide. I must leave the solution of this difficulty to some more advanced theory of waves.

I may appropriately close this section with a statement, in the form commonly used by nautical persons, of the most prominent effects of the diurnal tide at the several stations.

Assuming that the maximum diurnal tide, with positive sign for the semidiurnal high waters 1st division, occurred about July 9 and August 6, and with opposite sign on July 23, it appears that the maximum takes place when the moon's right ascension is about 9^h. This is not very accurate; first, because the solar diurnal tide is neglected; secondly, because the days adopted are not purely for maximum at semi-diurnal high water, but partly also refer to low water. Using however 9^h, it appears from the table in page 39, that the semidiurnal high water at Kilbaha follows the moon's Greenwich transit by 4^h 47^m; and, therefore, when the diurnal tide is greatest, the semidiurnal tide at Kilbaha occurs at 13^h 47^m Greenwich sidereal time, or 13^h 7^m Kilbaha sidereal time. If this happens at noon, the sun's right ascension

must be $13^h 7^m$, or the day must be about October 12; if it happens at six in the morning, the sun's right ascension must be $19^h 7^m$, and the day must be about January 6. In the same way the day may be found for other hours. The coefficient may be taken from the table in page 28, doubling the mean of the quantities in the three high water columns (without regard of sign) for the difference of two tides. Thus the following Table is formed.

Station.	Greatest difference of two high waters on same day.	Day when the excess of noon tide over midnight tide is greatest.	Day when the excess of morning tide over evening tide is greatest.	Station.	Greatest difference of two high waters on same day.	Day when the excess of noon tide over midnight tide is greatest.	Day when the excess of morning tide over evening tide is greatest.
	ft.				ft.		
Kilbaha	0·71	Oct. 12.	Jan. 6.	Ballycastle	1·16	Nov. 24.	Feb. 16.
Kilrush	0·55	Oct. 14.	Jan. 8.	Glenarm	1·48	Jan. 7.	April 10.
Foynes Island ..	0·72	Oct. 30.	Jan. 22.	Donaghadee	1·41	Jan. 13.	April 17.
Limerick	0·77	Nov. 7.	Jan. 30.	Ardglass	1·32	Jan. 13.	April 17.
Casleh Bay	0·65	Oct. 14.	Jan. 8.	Clogher Head. ...	1·29	Jan. 16.	April 20.
Galway	0·61	Oct. 17.	Jan. 10.	Kingstown	1·16	Jan. 19.	April 23.
Old Head	1·14	Oct. 17.	Jan. 10.	Dunmore East ..	0·44	Oct. 21.	Jan. 5.
Mullaghmore	1·65	Oct. 26.	Jan. 18.	New Ross	0·53	Nov. 2.	Jan. 24.
Buncrana	1·37	Nov. 5.	Jan. 28.	Passage West. ...	0·30	Oct. 21.	Jan. 14.
Port Rush	1·11	Nov. 14.	Feb. 3.	Castle Townsend..	0·08	Oct. 13.	Jan. 7.
Carrowkeel	1·31	Nov. 26.	Feb. 18.				

Section X.—*Method of expressing the height of the water, throughout every individual tide, by sines and cosines of arcs, and expressions in this form for every tide in the whole series of observations, except those at Courtown.*

The times of high water (and similarly those of low water) having had their principal irregularities smoothed down by the operations described in Section II., and being corrected for the diurnal equation in time ascertained by the operations of Section III., present a series of times, which are liable perhaps to something like constant error from the method involuntarily adopted by the computer in fixing on the time of high water, and which are affected by the peculiar form of the tidal function at each station, but which nevertheless follow at intervals equal (with very considerable accuracy) to the true tidal day of the place. This being understood, it will be seen that the following process entirely corrects any error of the supposed times of high or low water in its exhibition of the time of maximum of the first tidal argument, and is entirely free from the effects of such error in the exhibition of other quantities.

The whole number of observations, equidistant in time, made in the course of one tide, being about 150, if we divide this duration into sixteen equal parts we shall have at least nine observations in each part; and the mean time of these nine observations cannot in any case be more than $2\frac{1}{2}$ minutes from the middle of that part. It appears evident here that we may use the mean of all the heights in one portion to represent (with smaller error than unavoidably occurs in the observations) the

mean which would have been obtained if observations had been taken at infinitely small equal intervals. The same remark applies in a stronger degree if the whole duration be divided into twelve parts.

Let us use the term *phase* for an angle proportional to the time which increases by 360° in a complete tide; and let it be assumed that the height of the water can be expressed by the following formula:

$$A_0 + A_1 \sin \text{phase} + A_2 \sin 2 \text{ phase} + A_3 \sin 3 \text{ phase} + A_4 \sin 4 \text{ phase} \\ + B_1 \cos \text{phase} + B_2 \cos 2 \text{ phase} + B_3 \cos 3 \text{ phase} + B_4 \cos 4 \text{ phase},$$

and suppose that the complete tide, or 360° of phase, is divided into sixteen equal parts and the mean height in each part taken.

The mean height in the first part will be,

$$A_0 + \frac{8}{\pi} A_1 \left(\cos 0 - \cos \frac{\pi}{8} \right) + \frac{8}{2\pi} A_2 \left(\cos 0 - \cos \frac{2\pi}{8} \right) + \frac{8}{3\pi} A_3 \left(\cos 0 - \cos \frac{3\pi}{8} \right) + \frac{8}{4\pi} A_4 \left(\cos 0 - \cos \frac{4\pi}{8} \right) \\ + \frac{8}{\pi} B_1 \left(\sin \frac{\pi}{8} - \sin 0 \right) + \frac{8}{2\pi} B_2 \left(\sin \frac{2\pi}{8} - \sin 0 \right) + \frac{8}{3\pi} B_3 \left(\sin \frac{3\pi}{8} - \sin 0 \right) + \frac{8}{4\pi} B_4 \left(\sin \frac{4\pi}{8} - \sin 0 \right).$$

The mean height in the second part will be,

$$A_0 + \frac{8}{\pi} A_1 \left(\cos \frac{\pi}{8} - \cos \frac{2\pi}{8} \right) + \frac{8}{2\pi} A_2 \left(\cos \frac{2\pi}{8} - \cos \frac{4\pi}{8} \right) + \frac{8}{3\pi} A_3 \left(\cos \frac{3\pi}{8} - \cos \frac{6\pi}{8} \right) + \frac{8}{4\pi} A_4 \left(\cos \frac{4\pi}{8} - \cos \frac{8\pi}{8} \right) \\ + \frac{8}{\pi} B_1 \left(\sin \frac{2\pi}{8} - \sin \frac{\pi}{8} \right) + \frac{8}{2\pi} B_2 \left(\sin \frac{4\pi}{8} - \sin \frac{2\pi}{8} \right) + \frac{8}{3\pi} B_3 \left(\sin \frac{6\pi}{8} - \sin \frac{3\pi}{8} \right) + \frac{8}{4\pi} B_4 \left(\sin \frac{8\pi}{8} - \sin \frac{4\pi}{8} \right),$$

and so on.

Now if we group these in the following manner,

$$(1\text{st} + 5\text{th} + 9\text{th} + 13\text{th}) + (2\text{nd} + 6\text{th} + 10\text{th} + 14\text{th}) - (3\text{rd} + 7\text{th} + 11\text{th} + 15\text{th}) \\ - (4\text{th} + 8\text{th} + 12\text{th} + 16\text{th}),$$

the sum will be $\frac{32}{\pi} A_4$.

If we group them in the following manner,

$$(1\text{st} + 5\text{th} + 9\text{th} + 13\text{th}) - (2\text{nd} + 6\text{th} + 10\text{th} + 14\text{th}) - (3\text{rd} + 7\text{th} + 11\text{th} + 15\text{th}) \\ + (4\text{th} + 8\text{th} + 12\text{th} + 16\text{th}),$$

the sum will be $\frac{32}{\pi} B_4$.

If we unite the adjacent means and group them thus,

$$\overline{(1\text{st} + 2\text{nd} + 9\text{th} + 10\text{th})} + \overline{(3\text{rd} + 4\text{th} + 11\text{th} + 12\text{th})} - \overline{(5\text{th} + 6\text{th} + 13\text{th} + 14\text{th})} \\ - \overline{(7\text{th} + 8\text{th} + 15\text{th} + 16\text{th})},$$

the sum will be $\frac{32}{\pi} A_2$.

If we group them in this manner,

$$\overline{(1\text{st} + 2\text{nd} + 9\text{th} + 10\text{th})} - \overline{(3\text{rd} + 4\text{th} + 11\text{th} + 12\text{th})} - \overline{(5\text{th} + 6\text{th} + 13\text{th} + 14\text{th})} \\ + \overline{(7\text{th} + 8\text{th} + 15\text{th} + 16\text{th})},$$

the sum will be $\frac{32}{\pi} B_2$.

If we unite the adjacent numbers already formed by union, so as to have the sum of four adjacent means together, and combine them thus,

$$(1\text{st} + 2\text{nd} + 3\text{rd} + 4\text{th}) + (5\text{th} + 6\text{th} + 7\text{th} + 8\text{th}) - (9\text{th} + 10\text{th} + 11\text{th} + 12\text{th}) \\ - (13\text{th} + 14\text{th} + 15\text{th} + 16\text{th}),$$

the sum is $\frac{32}{\pi} A_1 + \frac{32}{3\pi} A_3$.

If we combine them in this manner,

$$(1\text{st} + 2\text{nd} + 3\text{rd} + 4\text{th}) - (5\text{th} + 6\text{th} + 7\text{th} + 8\text{th}) - (9\text{th} + 10\text{th} + 11\text{th} + 12\text{th}) \\ + (13\text{th} + 14\text{th} + 15\text{th} + 16\text{th}),$$

the sum is $\frac{32}{\pi} B_1 - \frac{32}{3\pi} B_3$.

Then if we divide the complete tide into twelve equal parts, and take the mean height in each, we shall have

Mean height in the first part =

$$A_0 + \frac{6}{\pi} A_1 \left(\cos 0 - \cos \frac{\pi}{6} \right) + \frac{6}{2\pi} A_2 \left(\cos 0 - \cos \frac{2\pi}{6} \right) + \frac{6}{3\pi} A_3 \left(\cos 0 - \cos \frac{3\pi}{6} \right) + \frac{6}{4\pi} A_4 \left(\cos 0 - \cos \frac{4\pi}{6} \right) \\ + \frac{6}{\pi} B_1 \left(\sin \frac{\pi}{6} - \sin 0 \right) + \frac{6}{2\pi} B_2 \left(\sin \frac{2\pi}{6} - \sin 0 \right) + \frac{6}{3\pi} B_3 \left(\sin \frac{3\pi}{6} - \sin 0 \right) + \frac{6}{4\pi} B_4 \left(\sin \frac{4\pi}{6} - \sin 0 \right),$$

and so on. And combining these in the following manner,

$$(1\text{st} + 5\text{th} + 9\text{th}) + (2\text{nd} + 6\text{th} + 10\text{th}) - (3\text{rd} + 7\text{th} + 11\text{th}) - (4\text{th} + 8\text{th} + 12\text{th}),$$

the sum will be $\frac{24}{\pi} A_3$, or the sum $\times \frac{4}{3} = \frac{32}{\pi} A_3$.

And if we combine them in the following manner;

$$(1\text{st} + 5\text{th} + 9\text{th}) - (2\text{nd} + 6\text{th} + 10\text{th}) - (3\text{rd} + 7\text{th} + 11\text{th}) + (4\text{th} + 8\text{th} + 12\text{th}),$$

the sum will be $\frac{24}{\pi} B_3$, or the sum $\times \frac{4}{3} = \frac{32}{\pi} B_3$.

By applying one-third of these to the expressions last found, we shall obtain $\frac{32}{\pi} A_1$ and $\frac{32}{\pi} B_1$.

The mean of all the means, either in the division by sixteen or in that by twelve, is A_0 .

The whole of these operations (after taking the means of the original observations) are performed with great facility, and without the possibility of mistake, by means of a printed skeleton form, of which a specimen will be given shortly.

The next thing to be considered is, how we shall correct these numbers for the effect of diurnal tide, which is included in the observations, but from which our formula for semidiurnal tide is to be freed. Suppose that the tide begins with high water, and suppose a to be the effect of diurnal tide at that high water, b the effect of diurnal tide at the low water following, or that which occurs in the middle of the tide. Then the complete effect of diurnal tide is represented by

$$a \cdot \cos \frac{\text{phase}}{2} + b \cdot \sin \frac{\text{phase}}{2};$$

and the question now is, how this function can be represented, through the course of one tide, by a formula similar to

$$A_0 + A_1 \sin \text{phase} + A_2 \sin 2 \text{phase} + \&c. \\ + B_1 \cos \text{phase} + B_2 \cos 2 \text{phase} + \&c.$$

For this purpose I have taken the mean value of the function for each sixteenth part, and for each twelfth part, of the entire circle of phase, and have combined these numbers according to the rules just laid down. The result is that

$$\text{The sum } \frac{32}{\pi} A_4 \text{ is increased by } \frac{16}{\pi} \times 0.3980 \times a.$$

$$\text{The sum } \frac{32}{\pi} B_4 \text{ is increased by } -\frac{16}{\pi} \times 0.0392 \times b.$$

$$\text{The sum } \frac{32}{\pi} A_2 \text{ is increased by } \frac{16}{\pi} \times 0.8284 \times a.$$

$$\text{The sum } \frac{32}{\pi} B_2 \text{ is increased by } -\frac{16}{\pi} \times 0.1648 \times b.$$

$$\text{The sum } \frac{32}{\pi} A_1 + \frac{32}{3\pi} A_3 \text{ is increased by } \frac{16}{\pi} \times 2a.$$

$$\text{The sum } \frac{32}{\pi} B_1 - \frac{32}{3\pi} B_3 \text{ is increased by } -\frac{16}{\pi} \times 0.8284 \times b.$$

$$\text{The sum } \frac{24}{\pi} A_3 \text{ is increased by } \frac{12}{\pi} \times 0.5360 \times a.$$

$$\text{The sum } \frac{24}{\pi} B_3 \text{ is increased by } -\frac{12}{\pi} \times 0.0704 \times b.$$

$$\text{The mean, or } A_0, \text{ is increased by } +\frac{2}{\pi} b.$$

The corrections are to be applied with opposite signs, in order to free from the effects of diurnal tide the results given by the observations.

Another cause for which a correction is due is, the difference of height at the beginning and at the end (the correction for diurnal tide being previously applied). If the whole rise c be supposed to have come by uniform degrees, the effects produced in the sums $\frac{32}{\pi} A_4$, $\frac{32}{\pi} A_2$, $\frac{32}{\pi} A_1 + \frac{32}{3\pi} A_3$, and $\frac{24}{\pi} A_3$, are $-c$, $-2c$, $-4c$, and $-c$. The corrections must have opposite signs.

In this manner (confining ourselves for a moment to the consideration of 4 phase) we have such expressions as $\frac{32}{\pi} A_4$ and $\frac{32}{\pi} B_4$. And the quantity which we wish to obtain is $A_4 \sin 4 \text{ phase} + B_4 \cos 4 \text{ phase}$, which may be converted into one of this form, $\sqrt{A_4^2 + B_4^2} \times \sin .4 \text{ phase} + \phi$, where $\tan \phi = \frac{B_4}{A_4}$. The coefficient is

$$= \frac{\pi}{32} \sqrt{\left(\frac{32}{\pi} A_4\right)^2 + \left(\frac{32}{\pi} B_4\right)^2}.$$

As, in the analysis of all the tides, this transformation was to be performed about 6000 times, it was highly important to devise an easy method of effecting it. For this purpose the following mechanical arrangement was contrived. Upon a nearly square piece of pasteboard were carefully traced two scales at right angles to each other, with graduations of equal parts proceeding from the point of union. Upon the edge of another narrow piece of pasteboard was traced a graduation whose parts were to the parts of the former in the proportion of 3·2 to 3·1416. The commencing point of this graduation was made the centre of a quadrant, of which one radius was in the line of graduation produced. The divisions of the quadrant, proceeding from the line of graduation, were marked from 0 to 90°, and also from 360° to 270°. The method of using it was; to insert a needle at the centre of the quadrant, and to plant its point upon one of the lines of the large pasteboard at the graduation corresponding to $\frac{32}{\pi} A_4$; then to plant a second needle in the other line of the large pasteboard at the graduation corresponding to $\frac{32}{\pi} B_4$, and to turn the graduated edge of the long piece till it touched this second needle; the reading of the graduated edge, with a shift of the decimal point, gave $\frac{\pi}{32} \sqrt{\left\{ \left(\frac{32}{\pi} A_4 \right)^2 + \left(\frac{32}{\pi} B_4 \right)^2 \right\}}$ or $\sqrt{A_4^2 + B_4^2}$; and the division of the quadrant cut by the straight line on the large pasteboard gave ϕ . When the signs of $\frac{32}{\pi} A_4$ and $\frac{32}{\pi} B_4$ are the same, the reading between 0 and 90° is to be taken; when different, that between 360° and 270° is to be taken; and in either case, when $\frac{32}{\pi} A_4$ is negative, 180° is to be added or subtracted.

Applying the same process to the four combinations of A_1 and B_1 , A_2 and B_2 , A_3 and B_3 , A_4 and B_4 , we have the height of the water at every instant expressed by the formula

$$A_0 + C_1 \sin(\text{phase} + \phi_1) + C_2 \sin(2 \text{ phase} + \phi_2) + C_3 \sin(3 \text{ phase} + \phi_3) + C_4 \sin(4 \text{ phase} + \phi_4),$$

where the *phase* is an angle which is measured from the assumed commencement of the tide, and may be converted into time by multiplying it by $\frac{\text{whole duration of tide}}{360^\circ}$. It is evident however that the argument ($\text{phase} + \phi_1$) commences at the time when the water would be at its mean height before attaining its greatest height, if the oscillation of surface were supposed to depend on that term only. The time of high water, on the same supposition, would be given by making $\text{phase} + \phi_1 = 90^\circ$, or $\text{phase} = 90^\circ - \phi_1$; converting the expression, when found, into time by the rule above. Or the time of low water, on the same supposition, would be given by making $\text{phase} + \phi_1 = 270^\circ$, or $\text{phase} = 270^\circ - \phi_1$. It is convenient to choose, of these two expressions, that which gives the smaller quantity. The quantity so found is to be added to the Greenwich time of assumed commencement of tide, and it gives the Greenwich time of high

water or low water, on the supposition that the fluctuation depends entirely on the argument (phase $+\phi_1$). Since the Theory of Waves, as applied to Tides, leads us to refer every angle to that argument, and induces us to suppose that the term connected with that argument is the only one which is immediately created by the tidal forces (the others depending numerically on it almost in the same way in which the coefficients of successive multiples of *anomaly* depend on that of the simple *anomaly*), it appears to be proper to consider the times of high and low water thus found as the genuine times of high and low water. For the sake of distinction I call them the *Analysed Times*.

As it is convenient to use the time of one phase only, I have, when the analysis gave the *Analysed Times* of two low waters, taken their mean for the *Analysed Time of High Water*.

Now if we put phase $+\phi_1=p$, $\phi_2-2\phi_1=c_2$, $\phi_3-3\phi_1=c_3$, $\phi_4-4\phi_1=c_4$, our expression for the height at every instant will be

$$A_0 + C_1 \sin p + C_2 \sin (2p + c_2) + C_3 \sin (3p + c_3) + C_4 \sin (4p + c_4),$$

and this, with a statement of the time at which the argument p has the value of 90° (or the *Analysed Time of High Water*), gives a complete knowledge of the form of every tide.

I annex a specimen of the printed skeleton form in which the calculations described in this section were made (the figures, and the words in italics, being inserted for each special tide). And I subjoin the whole of the results for the twenty-one stations. Each line is the digested result of about 170 observations.

Height of the Water in each individual tide at Kilbaha, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
.....
.....
.....
.....
June	24. 18 29	16·40	5·10	0·14	146	0·19	215	0·06	115
	25. 6 39								
	25. 18 59	15·79	5·05	0·13	199	0·09	318	0·01	40
	26. 7 13								
	26. 19 27	15·50	4·71	0·18	170	0·03	291	0·05	140
	27. 7 46								
	27. 20 0	15·65	4·20	0·08	158	0·07	204	0·09	82
	28. 8 20								
	28. 20 43	15·52	3·76	0·13	199	0·05	192	0·02	99
	29. 9 0								
	29. 21 32	15·65	3·25	0·10	210	0·08	244	0·02	55
	30. 10 2								
	30. 22 25	15·70	2·93	0·04	144	0·06	227	0·03	151
July	1. 10 57	15·51	2·73	0·06	216	0·08	231	0·02	52
	1. 23 29								
	2. 12 2	15·96	2·82	0·02	232	0·07	225	0·06	109
	3. 0 35								
	3. 13 11	16·41	3·36	0·12	212	0·07	281	0·04	55
	4. 1 46								
	4. 14 13	15·49	3·95	0·12	216	0·09	262	0·03	37
	5. 2 39								
	5. 15 6	15·43	4·73	0·10	187	0·09	288	0·04	230
	6. 3 32								
	6. 15 54	16·00	5·32	0·04	153	0·01	210	0·01	289
	7. 4 21								
	7. 16 42	16·05	5·86	0·03	225	0·09	255	0·04	88
	8. 5 0								
	8. 17 22	15·96	6·33	0·14	132	0·04	217	0·02	44
	9. 5 42								
	9. 18 16	15·98	6·80	0·17	142	0·14	248	0·08	61
	10. 6 40								
	10. 18 46	16·12	6·80	0·26	155	0·14	305	0·07	154
	11. 7 10								
	11. 19 39	15·79	6·48	0·20	175	0·02	121	0·07	140
	12. 8 2								
	12. 20 31	15·46	5·88	0·29	211	0·10	226	0·03	67
	13. 8 55								
	13. 21 21	15·26	4·97	0·17	200	0·09	220	0·02	122
	14. 9 48								
	14. 22 24	15·39	4·36	0·08	205	0·11	256	0·01	176
	15. 10 58								
	15. 23 32	16·01	3·72	0·19	196	0·06	328	0·08	228
	16. 12 10								
	17. 0 48	16·00	3·47	0·09	237	0·04	247	0·04	4
	17. 13 27								
	18. 2 5	15·80	3·70	0·15	238	0·11	265	0·04	207
	18. 14 36								
	19. 3 6	15·72	4·15	0·09	276	0·05	288	0·03	148
	19. 15 32								
	20. 3 59	15·76	4·48	0·03	152	0·07	215	0·05	210
	20. 16 20								
	21. 4 38	15·54	4·93	0·11	161	0·00	219	0·04	137
	21. 16 53								
	22. 5 9	15·56	5·35	0·08	182	0·02	350	0·04	174
	22. 17 28								
	23. 5 45	15·51	5·50	0·13	166	0·01	112	0·03	28

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .	
	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°	
July	1842.	18	2	15·47	5·55	0·14	152	0·05	226	0·02	28
	23.	6	18								
	24.	18	32								
	24.	6	52								
	25.	19	5								
	25.	7	22								
	26.	19	39								
	26.	7	55								
	27.	20	12								
	27.	8	27								
	28.	20	52								
	28.	9	11								
	29.	21	32								
	29.	9	59								
30.	22	26									
30.	11	6									
31.	23	46	15·45	2·57	0·06	182	0·05	199	0·02	338	
August	1.	12	27	15·75	2·90	0·03	218	0·03	76	0·02	132
	2.	1	8								
	2.	13	43								
	3.	2	18	15·38	3·62	0·01	261	0·05	240	0·06	60
	3.	14	45								
	4.	3	11	15·45	4·55	0·07	205	0·09	215	0·11	104
	4.	15	34								
	5.	3	54	15·68	5·30	0·03	192	0·10	218	0·04	197
	5.	16	18								
	6.	4	42	15·70	6·32	0·05	135	0·08	233	0·05	0
	6.	17	2								
	7.	5	22	15·80	7·30	0·21	170	0·04	7	0·04	184
	7.	17	39								
	8.	6	2	15·89	7·63	0·18	148	0·05	127	0·07	100
	8.	18	27								
	9.	6	48	15·94	7·62	0·32	168	0·03	266	0·05	91
	9.	19	10								
	10.	7	34	15·76	7·08	0·24	161	0·10	221	0·07	74
	10.	19	55								
	11.	8	21	15·63	6·12	0·29	168	0·06	193	0·05	71
	11.	20	45								
	12.	9	8	15·64	5·01	0·19	178	0·07	206	0·06	72
	12.	21	45								
13.	10	22	15·41	4·06	0·13	227	0·13	242	0·04	159	
13.	22	58									
14.	11	43	15·27	3·24	0·10	244	0·11	309	0·02	180	
15.	0	27	15·27	2·79	0·12	221	0·07	269	0·01	0	
15.	13	9									
16.	1	50	15·41	3·16	0·06	235	0·10	268	0·02	316	
16.	14	19									
17.	2	48	15·59	3·71	0·05	238	0·05	279	0·00	51	
17.	15	10									
18.	3	31	15·64	4·32	0·09	203	0·05	270	0·01	24	
18.	15	50									
19.	4	11	15·76	4·76	0·08	121	0·05	178	0·02	108	
19.	16	29									
20.	4	43	15·74	5·29	0·13	165	0·09	242	0·03	189	
20.	16	55									
21.	5	11	15·54	5·73	0·11	177	0·06	232	0·04	163	
21.	17	26									
22.	5	38	15·24	5·86	0·16	149	0·05	154	0·04	127	
22.	17	54									
23.	6	12	15·58	5·80	0·15	135	0·10	222	0·04	88	

Height of the Water in each individual tide at Kilrush, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
.....
.....
June	23. 17 50	17·14	5·90	0·17	111	0·07	284	0·07	167
	24. 6 11								
	24. 18 42	17·20	5·53	0·21	105	0·08	155	0·06	141
	25. 6 52								
	25. 19 14	16·62	5·47	0·12	119	0·05	261	0·11	175
	26. 7 28								
	26. 19 32	16·29	5·18	0·20	124	0·07	231	0·10	146
	27. 7 51								
	27. 20 23	16·40	4·63	0·16	150	0·05	153	0·04	107
	28. 8 43								
	28. 20 52	16·48	4·23	0·42	116	0·17	139	0·03	97
	29. 9 17								
	29. 21 41	16·36	3·80	0·16	106	0·05	194	0·05	130
	30. 10 7								
	30. 22 33	16·52	3·39	0·11	86	0·01	30	0·03	230
July	1. 11 4								
	1. 23 35	16·30	3·12	0·10	135	0·09	186	0·00	130
	2. 12 13								
	3. 0 50	16·74	3·22	0·10	84	0·06	208	0·01	240
	3. 13 21								
	4. 1 52	17·25	3·75	0·16	150	0·07	201	0·02	269
	4. 14 19								
	5. 2 45	16·51	4·50	0·12	136	0·09	237	0·01	333
	5. 14 58	16·20	5·13	0·15	314	0·17	177	0·08	23
	6. 3 26								
	6. 15 57	16·75	5·71	0·17	58	0·06	257	0·02	181
	7. 4 24								
	7. 16 52	16·90	6·30	0·11	80	0·11	258	0·06	134
	8. 5 10								
	8. 17 25	16·63	6·87	0·20	77	0·12	155	0·05	205
	9. 5 45								
	9. 18 9	16·86	7·17	0·31	99	0·09	254	0·13	184
	10. 6 33								
	10. 18 58	17·02	7·15	0·23	103	0·02	239	0·05	210
	11. 7 22								
	11. 19 40	16·70	6·82	0·17	100	0·04	76	0·05	119
	12. 8 3								
	12. 20 32	16·35	6·13	0·24	152	0·09	161	0·01	44
	13. 8 56								
	13. 21 21	16·00	5·43	0·18	125	0·06	194	0·03	179
	14. 9 48								
	14. 22 25	16·10	4·78	0·10	159	0·13	264	0·05	170
	15. 10 58								
	15. 23 30	16·63	4·18	0·18	151	0·07	267	0·02	318
	16. 12 11								
	17. 0 51	16·70	3·99	0·07	190	0·03	201	0·02	218
	17. 13 28								
	18. 2 4	16·60	4·11	0·11	140	0·09	241	0·04	201
	18. 14 36								
	19. 3 8	16·45	4·64	0·05	128	0·03	254	0·07	167
	19. 15 30	16·46	5·01	0·10	60	0·08	173	0·05	136
	20. 3 57								
	20. 16 24	16·24	5·43	0·13	101	0·04	112	0·03	167
	21. 4 42								
	21. 17 3	16·29	5·82	0·12	100	0·04	194	0·03	144
	22. 5 19								
	22. 17 39	16·28	6·02	0·20	111	0·04	179	0·03	193
	23. 5 56								

The value $c_3=314^\circ$ for July 5 and 6 is correct.

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p = 90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
July	23.	18 12	16.31	6.02	0.21	120	0.09	228	0.05	205
	24.	6 28	16.36	5.87	0.26	100	0.06	184	0.03	123
	24.	18 40								
	25.	7 0	16.21	5.69	0.26	122	0.04	206	0.07	146
	25.	19 15								
	26.	7 32	15.98	5.29	0.15	153	0.02	211	0.01	95
	26.	19 50								
	27.	8 6	15.95	4.82	0.19	129	0.07	165	0.08	146
	27.	20 18								
	28.	8 33	15.80	4.20	0.20	143	0.06	204	0.02	259
	28.	20 56								
	29.	9 15	16.02	3.72	0.23	118	0.10	215	0.02	119
	29.	21 33								
	30.	10 6	15.98	3.10	0.09	142	0.04	224	0.01	253
	30.	22 38								
	31.	11 15	16.14	2.89	0.12	117	0.09	210	0.01	29
	31.	23 51								
August	1.	12 31	16.59	3.33	0.11	143	0.06	226	0.04	206
	2.	1 10	16.32	4.15	0.16	124	0.08	223	0.02	213
	2.	13 52								
	3.	2 33	16.34	4.89	0.13	117	0.06	234	0.03	266
	3.	15 7								
	4.	3 35	16.64	5.86	0.11	115	0.03	285	0.10	185
	4.	15 57								
	5.	4 17	16.68	6.87	0.22	82	0.07	195	0.06	151
	5.	16 42								
	6.	5 6	16.80	7.59	0.25	71	0.11	219	0.06	128
	6.	17 30								
	7.	5 50	16.76	7.96	0.34	91	0.10	208	0.08	199
	7.	18 9								
	8.	6 32	16.69	7.95	0.25	116	0.11	217	0.03	231
	8.	18 57								
	9.	7 18	16.51	7.49	0.24	113	0.10	198	0.02	123
	9.	19 38								
	10.	8 2	16.36	6.62	0.29	127	0.16	197	0.07	78
	10.	20 23								
	11.	8 49	16.38	5.50	0.15	138	0.11	183	0.06	116
	11.	21 12								
	12.	9 35	16.17	4.58	0.11	150	0.11	213	0.07	175
	12.	22 7								
	13.	10 41	15.98	3.76	0.13	148	0.08	240	0.07	212
	13.	23 14								
	14.	11 59	16.07	3.29	0.12	153	0.06	235	0.02	284
	15.	0 43								
	15.	13 25	16.24	3.69	0.10	148	0.05	265	0.03	190
	16.	2 7								
	16.	14 37	16.53	4.30	0.11	117	0.03	123	0.01	231
	17.	3 7								
	17.	15 28	16.39	4.70	0.15	31	0.15	208	0.08	117
	18.	3 50								
	18.	16 10	16.55	5.33	0.14	84	0.11	170	0.01	209
	19.	4 31								
	19.	16 54	16.53	5.80	0.24	112	0.06	216	0.07	164
	20.	5 8								
	20.	17 16	16.31	6.20	0.13	60	0.08	211	0.03	65
	21.	5 32								
	21.	17 50	16.00	6.32	0.21	109	0.12	150	0.04	130
	22.	6 2								
	22.	18 13	16.42	6.36	0.18	68	0.14	176	0.08	72
	23.	6 31								

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p = 90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
		ft.	ft.	ft.	°	ft.	°	ft.	°
1842.	h m								
July	23. 18 55	16·99	6·89	0·61	113	0·21	172	0·02	10
	24. 7 11								
	24. 19 23	17·06	6·80	0·60	115	0·18	159	0·04	306
	25. 7 43								
	25. 19 55	16·93	6·50	0·61	116	0·22	160	0·02	307
	26. 8 12								
	26. 20 25	16·77	6·11	0·58	123	0·16	159	0·00	202
	27. 8 41								
	27. 20 54	16·78	5·80	0·55	110	0·15	152	0·05	92
	28. 9 11								
	28. 21 27	16·60	5·19	0·47	110	0·10	139	0·06	135
	29. 9 47								
	29. 22 6	16·78	4·52	0·46	113	0·09	186	0·01	210
	30. 10 33								
	30. 23 0	16·68	3·86	0·35	119	0·06	173	0·04	61
	31. 11 39								
August	1. 0 17	16·89	3·64	0·39	127	0·13	190	0·03	335
	1. 13 1	17·20	4·07	0·38	137	0·12	198	0·05	304
	2. 1 45								
	2. 14 24	17·00	4·89	0·44	131	0·12	189	0·03	269
	3. 3 2								
	3. 15 37	17·16	5·66	0·34	115	0·13	171	0·00	122
	4. 4 7								
	4. 16 37	17·38	6·59	0·43	121	0·22	165	0·02	287
	5. 4 57								
	5. 17 27	17·38	7·71	0·55	107	0·29	160	0·01	35
	6. 5 51								
	6. 18 17	17·53	8·29	0·64	102	0·37	162	0·04	259
	7. 6 37								
	7. 18 57	17·57	8·58	0·74	112	0·45	165	0·11	249
	8. 7 20								
	8. 19 45	17·56	8·51	0·72	114	0·39	171	0·09	283
	9. 8 6								
	9. 20 25	17·48	8·12	0·71	114	0·34	167	0·09	261
	10. 8 49								
	10. 21 7	17·13	7·56	0·64	111	0·28	167	0·01	77
	11. 9 28								
	11. 21 48	17·22	6·66	0·59	118	0·25	159	0·04	292
	12. 10 14								
	12. 22 39	16·93	5·40	0·40	122	0·13	166	0·03	77
	13. 11 9								
	13. 23 39	16·72	4·40	0·33	140	0·05	183	0·00	133
	14. 12 23								
	15. 1 7	16·72	3·91	0·36	140	0·12	209	0·02	337
	15. 13 50								
	16. 2 33	16·77	4·30	0·32	145	0·09	204	0·03	356
	16. 15 9								
	17. 3 39	17·06	4·80	0·33	124	0·08	177	0·01	175
	17. 16 5								
	18. 4 30	17·11	5·47	0·41	115	0·11	173	0·02	259
	18. 16 51								
	19. 5 12	17·28	6·15	0·50	113	0·19	163	0·02	109
	19. 17 35								
	20. 5 49	17·25	6·60	0·47	106	0·22	181	0·04	307
	20. 18 3								
	21. 6 19	17·04	6·99	0·60	109	0·23	167	0·04	54
	21. 18 35								
	22. 6 47	16·73	7·10	0·57	111	0·24	175	0·03	350
	22. 19 0								
	23. 7 18	17·10	7·06	0·59	111	0·30	167	0·02	22

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p = 90^\circ$.		A_0	C_1	C_2	c_2	C_3	c_3	C_4	c_4
		ft.	ft.	ft.	°	ft.	°	ft.	°
1842. July	23. 19 31	17.55	8.40	0.99	85	0.76	138	0.48	163
	24. 7 47								
	24. 20 8	17.42	8.50	0.97	76	0.83	111	0.52	138
	25. 8 23	17.34	8.32	0.94	75	0.70	111	0.46	144
	25. 20 37								
	26. 8 51	17.10	7.87	0.95	85	0.67	113	0.46	147
	26. 21 5								
	27. 9 18	17.19	7.31	1.03	88	0.74	122	0.29	160
	27. 21 30								
	28. 9 43	16.86	6.71	1.11	91	0.59	115	0.40	183
	28. 21 55								
	29. 10 14	17.21	5.72	0.97	96	0.51	135	0.17	165
29. 22 33									
30. 10 56	16.94	4.95	0.75	110	0.29	142	0.06	214	
30. 23 19									
31. 11 57	17.11	4.60	0.76	119	0.24	145	0.15	224	
August 1. 0 35									
1. 13 31	17.44	4.31	0.73	113	0.35	165	0.16	264	
2. 2 26									
2. 14 48	17.12	5.88	0.75	132	0.39	160	0.08	186	
3. 3 25									
3. 16 3	17.65	6.81	0.63	105	0.49	143	0.31	185	
4. 4 38									
4. 17 10	18.09	8.10	0.81	86	0.72	148	0.35	171	
5. 5 30									
5. 18 5	18.02	9.30	1.09	75	0.81	127	0.51	146	
6. 6 29									
6. 18 50	18.41	9.62	1.17	67	0.94	115	0.55	144	
7. 7 10									
7. 19 45	17.99	10.38	0.99	35	0.74	101	0.64	117	
8. 8 8									
8. 20 30	18.05	10.36	0.93	40	0.77	100	0.66	118	
9. 8 48									
9. 21 6	18.16	9.95	0.96	52	0.91	107	0.69	122	
10. 9 27									
10. 21 47	17.73	9.17	0.91	64	0.90	118	0.63	129	
11. 10 5									
11. 22 23	17.77	8.24	0.88	85	0.89	122	0.42	152	
12. 10 43									
12. 23 3	17.18	6.89	0.87	107	0.62	139	0.22	146	
13. 11 31									
13. 23 58	16.87	5.56	0.79	130	0.37	169	0.09	201	
14. 12 43									
15. 1 28	16.86	4.95	0.67	134	0.28	160	0.09	210	
15. 14 8									
16. 2 47	16.85	5.50	0.75	125	0.39	165	0.13	204	
16. 15 29									
17. 3 56	17.20	6.08	0.65	109	0.37	161	0.10	141	
17. 16 29									
18. 4 56	17.33	6.87	0.80	106	0.46	143	0.24	195	
18. 17 20									
19. 5 41	17.73	7.64	0.94	96	0.62	135	0.30	195	
19. 18 7									
20. 6 21	17.84	8.05	1.10	89	0.79	143	0.38	172	
20. 18 36									
21. 6 52	17.58	8.49	1.03	80	0.74	133	0.49	150	
21. 19 9									
22. 7 21	17.18	8.71	0.98	76	0.74	118	0.51	126	
22. 19 35									
23. 7 53	17.63	8.52	1.12	78	0.82	131	0.43	150	

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
		ft.	ft.	ft.	°	ft.	°	ft.	°
July	1842. h m								
	23. 18 2	17·93	5·70	0·21	154	0·08	230	0·03	289
	24. 6 18								
	24. 18 32	17·86	5·66	0·19	141	0·05	208	0·07	329
	25. 6 52								
	25. 19 7	17·71	5·37	0·21	163	0·06	258	0·04	12
	26. 7 24								
	26. 19 41	17·56	4·97	0·18	176	0·07	196	0·03	329
	27. 7 57								
	27. 20 13	17·57	4·44	0·16	193	0·08	191	0·06	296
	28. 8 28								
	28. 20 50	17·44	3·75	0·18	179	0·04	196	0·05	313
	29. 9 9								
	29. 21 28	17·62	3·35	0·08	157	0·07	233	0·04	318
30. 9 59									
30. 22 29	17·50	2·79	0·05	240	0·04	170	0·05	84	
31. 11 8									
31. 23 47	17·71	2·64	0·06	190	0·09	240	0·04	17	
August	1. 12 30								
	2. 1 12	18·10	2·87	0·09	25	0·12	244	0·03	67
	2. 13 49								
	3. 2 26	17·73	3·70	0·04	204	0·09	271	0·03	88
	3. 14 53								
	4. 3 19	17·92	4·80	0·08	206	0·16	258	0·01	333
	4. 15 44								
	5. 4 4	18·17	5·52	0·05	259	0·08	246	0·08	34
	5. 16 30								
	6. 4 54	18·15	6·66	0·18	168	0·23	187	0·05	91
	6. 17 12								
	7. 5 32	18·29	7·58	0·17	121	0·21	206	0·03	331
	7. 17 52								
	8. 6 15	18·32	7·86	0·29	137	0·14	208	0·13	343
	8. 18 39								
	9. 7 0	18·18	7·90	0·33	172	0·14	226	0·09	11
	9. 19 21								
	10. 7 45	18·08	7·27	0·28	159	0·18	206	0·14	333
	10. 20 3								
	11. 8 29	17·96	6·30	0·29	169	0·19	198	0·12	337
	11. 20 56								
	12. 9 19	17·99	5·16	0·19	194	0·14	192	0·05	348
	12. 21 51								
13. 10 30	17·83	4·14	0·05	151	0·08	232	0·09	359	
13. 23 9									
14. 11 55	17·58	3·26	0·11	275	0·13	290	0·08	90	
15. 0 40									
15. 13 21	17·64	2·90	0·06	217	0·05	241	0·05	93	
16. 2 2									
16. 14 32	17·73	3·28	0·09	268	0·10	282	0·02	297	
17. 3 2									
17. 15 23	17·95	3·81	0·05	254	0·01	268	0·02	350	
18. 3 48									
18. 16 3	17·99	4·29	0·05	117	0·06	219	0·02	238	
19. 4 24									
19. 16 44	18·12	4·88	0·08	149	0·13	167	0·04	302	
20. 4 58									
20. 17 8	18·12	5·47	0·16	166	0·11	244	0·04	338	
21. 5 24									
21. 17 40	17·82	5·87	0·08	150	0·08	236	0·06	8	
22. 5 52									
22. 18 8	17·54	6·02	0·11	166	0·07	200	0·08	352	
23. 6 26									
		17·91	6·00	0·14	155	0·11	248	0·08	322

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .																		
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°																		
July	23.	18	5	17·33	6·01	0·20	162	0·22	208	0·11	331																	
	24.	6	21																									
	24.	18	34																									
	25.	6	54																									
	25.	19	7																									
	26.	7	25																									
	26.	19	33																									
	27.	7	49																									
August	27.	20	12	16·98	4·65	0·18	193	0·18	182	0·05	340																	
	28.	8	27																									
	29.	21	33									17·09	3·50	0·21	162	0·18	239	0·14	312									
	30.	10	6									17·04	2·87	0·06	241	0·09	239	0·14	56									
	30.	22	39																									
	31.	11	20									17·25	2·72	0·03	284	0·08	213	0·02	15									
	1.	0	0																									
	1.	12	39																									
	2.	1	18																									
		2.	13									50	17·66	3·12	0·05	128	0·13	243	0·02	321								
		3.	2									22																
		3.	14									48																
		4.	3									14																
		5.	16									26									18·05	6·84	0·16	150	0·23	187	0·11	1
		6.	4									50																
		6.	17									18																
		7.	5									38																
		7.	17									59																
		8.	6									22																
		8.	18									51																
	9.	7	12																									
	9.	19	34																									
	10.	7	58																									
10.	20	13	17·62	7·65	0·29	160	0·30	216	0·26	7																		
11.	8	39																										
12.	22	3									17·56	6·50	0·40	168	0·29	199	0·21	342										
13.	10	43																										
13.	23	22																										
14.	12	7																										
15.	0	52																										
15.	13	33																										
16.	2	14																										
17.	15	35																	17·21	4·52	0·09	166	0·22	200	0·04	83		
18.	3	59																										
18.	16	15																										
19.	4	36																										
19.	16	51																										
20.	5	5																										
20.	17	15																										
21.	5	31																										
21.	17	43																										
22.	5	55																										
22.	18	6	17·20	3·43	0·13	259	0·11	276	0·09	124																		
23.	6	24																										
	15.	0	52	17·12	3·05	0·02	27	0·05	198	0·10	116																	
	15.	13	33																									
	16.	2	14																									
	17.	15	35									17·23	3·38	0·03	234	0·16	290	0·05	314									
	18.	3	59																									
	18.	16	15																									
	19.	4	36																									
	19.	16	51																									
	20.	5	5																									
	20.	17	15																									
	21.	5	31																									
21.	17	43																										
22.	5	55																										
22.	18	6	17·43	4·46	0·02	154	0·11	188	0·09	291																		
23.	6	24																										
	17.	15	35	17·79	5·18	0·16	127	0·14	175	0·18	277																	
	18.	3	59																									
	18.	16	15																									
	19.	4	36																									
	19.	16	51																									
	20.	5	5																									
	20.	17	15																									
	21.	5	31																									
	21.	17	43																									
	22.	5	55																									
	22.	18	6									17·66	5·67	0·18	160	0·17	209	0·09	53									
23.	6	24																										
	17.	15	35	17·37	6·18	0·13	154	0·07	213	0·10	330																	
	18.	3	59																									
	18.	16	15																									
	19.	4	36																									
	19.	16	51																									
	20.	5	5																									
	20.	17	15																									
	21.	5	31																									
	21.	17	43																									
	22.	5	55																									
	22.	18	6									17·06	6·30	0·19	163	0·11	159	0·25	348									
23.	6	24																										
	17.	15	35	17·49	6·22	0·26	148	0·11	204	0·27	313																	
	18.	3	59																									
	18.	16	15																									
	19.	4	36																									
	19.	16	51																									
	20.	5	5																									
	20.	17	15																									
	21.	5	31																									
	21.	17	43																									
	22.	5	55																									
	22.	18	6																									

Height of the Water in each individual tide at Old Head, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
June	22. 17 15	18·51	5·11	0·12	133	0·08	331	0·08	298
	23. 5 39								
	23. 18 3	18·55	5·04	0·16	146	0·09	23	0·05	315
	24. 6 23								
	24. 18 40	18·49	4·73	0·12	207	0·17	222	0·09	44
	25. 6 50								
	25. 19 13	17·88	4·69	0·08	186	0·07	197	0·07	185
	26. 7 27								
	26. 20 3	17·53	4·40	0·12	192	0·03	28	0·08	342
	27. 8 22								

	28. 21 27	17·73	3·58	0·09	356	0·06	174	0·02	116
	29. 9 50								
	29. 22 12	17·65	3·07	0·06	348	0·09	160	0·03	60
	30. 10 32								
	30. 22 52	17·91	2·68	0·04	276	0·12	173	0·02	328
July	1. 11 25								
	1. 23 58	17·69	2·52	0·08	207	0·09	195	0·05	210
	2. 12 35								
	3. 1 12	18·02	2·65	0·05	98	0·06	230	0·04	244
	3. 13 43								
	4. 2 14	18·68	3·08	0·05	218	0·10	231	0·02	39
	4. 14 42								
	5. 3 9	17·86	3·71	0·12	161	0·15	280	0·04	57
	5. 15 21	17·43	4·12	0·19	327	0·16	293	0·15	220
	6. 3 55								

	7. 17 12	18·05	5·43	0·09	332	0·21	213	0·07	339
	8. 5 30								
	8. 17 52	18·23	6·12	0·13	93	0·14	222	0·09	316
	9. 6 12								
	9. 18 36	18·22	6·52	0·21	112	0·15	159	0·11	276
	10. 7 0								
	10. 19 18	18·15	6·29	0·12	194	0·17	186	0·05	303
	11. 7 42								
	11. 20 7	17·93	6·13	0·15	233	0·08	358	0·02	335
	12. 8 30								
	12. 20 56	17·83	5·41	0·16	167	0·01	128	0·03	245
	13. 9 20								
	13. 21 51	17·31	4·93	0·08	264	0·02	169	0·08	278
	14. 10 19								
	14. 22 47	17·52	4·07	0·10	206	0·08	270	0·06	267
	15. 11 22								
	15. 23 56	18·10	3·47	0·10	273	0·15	314	0·03	175
	16. 12 32								
	17. 1 7	18·08	3·21	0·16	249	0·25	275	0·11	215
	17. 13 44								
	18. 2 21	17·95	3·37	0·11	178	0·07	251	0·04	356
	18. 14 52								
	19. 3 23	17·88	3·91	0·02	326	0·16	289	0·07	21
	19. 15 44								
	20. 4 11	17·83	4·18	0·08	10	0·17	237	0·05	167
	20. 16 35								
	21. 4 53	17·61	4·59	0·04	120	0·07	279	0·04	113
	21. 17 14								
	22. 5 30	17·56	4·93	0·07	201	0·11	292	0·07	227
	22. 17 44								
	23. 6 1	17·61	5·10	0·05	112	0·07	272	0·06	311

Height of the Water in each individual tide at Mullaghmore, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
June	22. 17 45	18.44	4.98	0.17	246	0.04	72	0.08	189
	23. 5 45								
	23. 18 17	18.85	4.97	0.18	187	0.14	65	0.13	237
	24. 6 39								
	24. 18 46	19.12	4.77	0.17	27	0.25	274	0.33	108
	25. 6 56								
	25. 19 34	18.29	4.51	0.19	265	0.18	16	0.07	328
	26. 7 48								
	26. 20 10	18.01	4.10	0.13	157	0.15	30	0.10	261
	27. 8 29								
	27. 20 50	18.20	3.77	0.04	228	0.11	60	0.02	175
	28. 9 7								
	28. 21 24	18.25	3.32	0.16	115	0.12	327	0.07	282
	29. 9 49								
	29. 22 14	18.08	2.90	0.11	126	0.10	283	0.06	131
	30. 10 40								
	30. 23 6	18.33	2.56	0.09	224	0.02	244	0.12	204
July	1. 11 51								
	2. 0 36	18.46	2.51	0.09	95	0.12	234	0.05	50
	2. 13 3								
	3. 1 29	18.35	2.46	0.04	178	0.08	160	0.05	315
	3. 14 8								
	4. 2 46	18.97	2.87	0.22	358	0.10	151	0.05	18
	4. 15 3								
	5. 3 21	18.26	3.35	0.11	321	0.13	250	0.07	22
	5. 15 35								
	6. 4 19	17.96	3.88	0.04	149	0.04	126	0.05	265
	6. 16 37								
	7. 5 4	18.87	4.27	0.26	129	0.19	355	0.04	264
	7. 17 18								
	8. 5 36	18.42	5.11	0.33	291	0.19	275	0.11	171
	8. 17 59								
	9. 6 19	18.73	5.77	0.10	172	0.09	28	0.12	173
	9. 18 53								
	10. 7 17	18.65	6.02	0.22	166	0.19	16	0.08	260
	10. 19 30								
	11. 7 54	18.57	5.89	0.13	148	0.05	64	0.10	238
	11. 20 18								
	12. 8 41	18.01	5.68	0.15	204	0.17	56	0.06	188
	12. 21 10								
	13. 9 38	18.04	5.41	0.05	333	0.24	33	0.06	44
	13. 22 6								
	14. 10 32	17.61	4.59	0.17	241	0.13	45	0.08	204
	14. 22 57								
	15. 11 32	17.73	3.92	0.11	156	0.16	121	0.15	212
	16. 0 6								
	16. 12 44	18.15	3.16	0.25	276	0.12	152	0.09	246
	17. 1 22								
	17. 14 5	18.27	3.12	0.21	158	0.10	81	0.04	151
	18. 2 47								
	18. 15 17	18.22	3.15	0.09	176	0.04	68	0.02	112
	19. 3 46								
	19. 16 12	18.05	3.40	0.10	274	0.14	58	0.12	56
	20. 4 39								
	20. 16 57	18.09	3.73	0.14	254	0.12	227	0.05	199
	21. 5 15								
	21. 17 42	18.03	4.21	0.01	128	0.04	75	0.02	51
	22. 5 58								
	22. 18 3	17.68	4.50	0.30	223	0.17	217	0.17	176
	23. 6 20								
		17.87	4.85	0.07	51	0.13	46	0.07	192

Height of the Water in each individual tide at Buncrana, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
June	22. 18 2	17·23	5·28	0·18	184	0·12	263	0·04	113
	23. 6 18								
	23. 18 38	18·38	5·10	0·16	145	0·19	234	0·06	158
	24. 6 51								
	24. 19 23	18·22	5·10	0·12	215	0·16	266	0·05	259
	25. 7 38								
	25. 20 3	17·93	4·69	0·11	123	0·12	263	0·03	158
	26. 8 20								
	26. 20 35	17·44	4·37	0·20	161	0·09	281	0·05	282
	27. 9 7								
	27. 21 15	17·66	3·69	0·13	203	0·15	248	0·07	230
	28. 9 20								
	28. 22 0	17·47	3·22	0·09	164	0·11	256	0·00	74
	29. 10 25								
	29. 22 48	17·40	2·97	0·15	54	0·03	203	0·04	143
	30. 11 17								
	30. 23 46	17·71	2·64	0·15	79	0·04	244	0·03	203
July	1. 12 20								
	2. 0 54	17·52	2·42	0·12	100	0·07	243	0·02	260
	2. 13 32								
	3. 2 10	17·70	2·58	0·12	128	0·01	264	0·01	322
	3. 14 44								
	4. 3 18	18·18	3·04	0·17	186	0·11	322	0·02	97
	4. 15 48								
	5. 4 3	17·79	3·39	0·22	261	0·05	199	0·06	159
	5. 16 26								
	6. 5 4	17·06	4·18	0·08	280	0·05	300	0·03	279
	6. 17 24								
	7. 5 37	17·55	4·94	0·08	289	0·09	252	0·04	167
	7. 18 4								
	8. 6 29	17·70	5·75	0·10	193	0·05	283	0·03	221
	8. 18 52								
	9. 7 16	17·78	6·25	0·23	153	0·17	261	0·04	340
	9. 19 33								
	10. 8 0	17·85	6·57	0·28	163	0·11	322	0·09	296
	10. 20 21								
	11. 8 45	17·98	6·56	0·29	152	0·08	237	0·11	253
	11. 21 11								
	12. 9 29	17·32	6·10	0·24	177	0·04	253	0·11	328
	12. 22 5	17·31	5·79	0·26	215	0·15	265	0·07	241
	13. 10 33								
	13. 23 0	16·99	4·91	0·15	253	0·13	296	0·07	290
	14. 11 31								
	15. 0 2	16·81	4·06	0·13	227	0·02	359	0·02	208
	15. 12 38								
	16. 1 13	17·31	3·31	0·09	204	0·07	320	0·02	132
	16. 13 55								
	17. 2 36	17·53	3·15	0·07	177	0·03	263	0·05	143
	17. 15 15								
	18. 3 54	17·28	3·32	0·21	211	0·03	162	0·00	29
	18. 16 22								
	19. 5 2	17·29	3·70	0·07	269	0·02	26	0·04	287
	19. 17 19								
	20. 5 43	17·28	4·12	0·01	78	0·06	241	0·03	325
	20. 18 5								
	21. 6 22	17·16	4·61	0·10	137	0·03	311	0·06	207
	21. 18 39								
	22. 6 56	17·04	5·02	0·09	159	0·08	246	0·04	238
	22. 19 10								
	23. 7 24	17·16	5·21	0·12	106	0·08	256	0·05	345

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
July	23.	19 41	17.21	5.23	0.12	149	0.07	216	0.08	264
	24.	7 53	17.26	5.11	0.14	118	0.09	224	0.03	259
	24.	20 3								
	25.	8 20	16.96	4.87	0.05	108	0.07	194	0.01	325
	25.	20 37								
	26.	8 44	16.87	4.41	0.03	135	0.06	213	0.02	306
	26.	21 3								
	27.	9 23	17.04	3.87	0.05	159	0.07	217	0.02	266
	27.	21 38								
	28.	9 51	16.82	3.53	0.04	311	0.13	247	0.04	235
	28.	22 9								
	29.	10 29	17.02	2.86	0.11	58	0.08	234	0.02	179
	29.	22 49								
30.	11 18	17.06	2.41	0.12	27	0.06	190	0.02	15	
30.	23 46									
31.	12 31	17.24	2.25	0.14	126	0.01	284	0.01	95	
August	1.									1 16
	1.	14 1	17.68	2.65	0.18	189	0.10	225	0.05	234
	2.	2 45								
	2.	15 9	17.24	3.10	0.14	227	0.04	216	0.03	53
	3.	3 45								
	3.	16 13	17.60	3.87	0.05	292	0.09	195	0.05	199
	4.	4 36								
	4.	17 3	17.79	5.20	0.14	249	0.08	233	0.08	272
	5.	5 23								
	5.	17 43	17.77	6.20	0.12	210	0.06	102	0.14	294
	6.	6 4								
	6.	18 27	17.82	6.90	0.10	189	0.06	258	0.02	237
	7.	6 42								
	7.	19 6	17.67	7.42	0.25	151	0.34	280	0.08	56
	8.	7 31								
	8.	19 49	17.71	7.40	0.25	178	0.11	285	0.12	309
	9.	8 11								
	9.	20 32	17.71	6.68	0.26	168	0.12	300	0.08	313
	10.	8 53								
	10.	21 16	17.37	5.76	0.29	158	0.09	293	0.10	294
	11.	9 37								
	11.	22 16	17.22	4.92	0.03	254	0.06	328	0.03	310
	12.	10 43								
12.	23 10	17.32	3.62	0.10	244	0.09	266	0.04	313	
13.	11 50									
14.	0 30	17.09	2.76	0.08	146	0.06	325	0.03	154	
14.	13 17									
15.	2 3	16.96	2.50	0.13	208	0.04	69	0.01	315	
15.	14 43									
16.	3 22	16.99	2.81	0.11	217	0.03	90	0.02	96	
16.	15 53									
17.	4 18	17.46	3.32	0.01	172	0.06	244	0.04	165	
17.	16 48									
18.	5 4	17.37	3.88	0.01	169	0.03	196	0.03	209	
18.	17 22									
19.	5 39	17.54	4.52	0.09	161	0.07	273	0.02	9	
19.	17 52									
20.	6 4	17.70	5.04	0.01	202	0.06	185	0.05	235	
20.	18 22									
21.	6 37	17.33	5.38	0.17	156	0.08	213	0.06	237	
21.	18 49									
22.	7 4	17.11	5.50	0.17	129	0.08	266	0.09	330	
22.	19 15									
23.	7 36	17.67	5.46	0.06	148	0.05	292	0.05	302	

Height of the Water in each individual tide at Port Rush, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
June	22. 18 35	17·91	2·38	0·05	195	0·10	266	0·04	163
	23. 6 51								
	23. 19 3	18·62	2·22	0·03	229	0·14	251	0·05	132
	24. 7 16								
	24. 19 37	18·64	2·21	0·06	185	0·14	250	0·03	342
	25. 7 52								
	25. 20 16	18·26	1·89	0·05	141	0·12	267	0·03	323
	26. 8 33								
	26. 20 21	17·66	1·62	0·08	154	0·08	248	0·02	313
	27. 8 56								
	27. 21 43	17·92	1·39	0·11	221	0·12	239	0·01	171
	28. 9 48								
	28. 22 30	17·52	1·20	0·09	255	0·10	247	0·02	117
	29. 10 55								
	29. 23 30	17·41	1·03	0·08	324	0·12	290	0·04	226
	30. 12 30								
July	1. 0 39	17·95	0·87	0·07	84	0·05	310	0·00	156
	1. 13 23								
	2. 2 7	17·86	0·94	0·14	161	0·06	310	0·02	106
	2. 14 58								
	3. 3 48	17·93	1·13	0·11	172	0·04	338	0·01	66
	3. 16 12								
	4. 4 36	18·46	1·38	0·15	215	0·07	315	0·02	117
	4. 16 57								
	5. 5 18	18·10	1·57	0·12	239	0·11	274	0·05	155
	5. 17 8	17·35	1·76	0·06	223	0·12	252	0·04	104
	6. 5 46								
	6. 17 49	17·88	2·26	0·12	283	0·10	243	0·03	176
	7. 6 2								
	7. 18 27	18·00	2·60	0·06	193	0·15	274	0·02	150
	8. 6 52								
	8. 19 0	18·15	2·80	0·15	164	0·07	270	0·01	314
	9. 7 24								
	9. 19 38	18·28	2·90	0·14	158	0·14	265	0·02	334
	10. 8 5								
	10. 20 28	18·29	2·91	0·15	155	0·16	258	0·02	162
	11. 8 51								
	11. 21 12	17·83	2·58	0·18	143	0·11	260	0·01	355
	12. 9 30								
	12. 21 52	18·15	2·17	0·22	153	0·11	227	0·01	56
	13. 10 18								
	13. 22 47	17·37	1·72	0·21	150	0·09	254	0·02	105
	14. 11 25								
	15. 0 29	17·04	1·52	0·14	282	0·09	314	0·04	158
	15. 13 12								
	16. 1 55	17·52	1·21	0·07	276	0·05	335	0·04	169
	16. 14 39								
	17. 3 23	17·73	1·31	0·06	237	0·03	322	0·06	206
	17. 15 58								
	18. 4 32	17·51	1·41	0·14	227	0·08	285	0·01	156
	18. 16 57								
	19. 5 22	17·41	1·68	0·13	277	0·06	312	0·02	189
	19. 17 40								
	20. 6 4	17·56	1·82	0·01	29	0·06	274	0·02	176
	20. 18 15								
	21. 6 32	17·57	2·00	0·05	69	0·13	287	0·00	180
	21. 18 46								
	22. 7 23	17·47	2·14	0·02	56	0·12	275	0·01	253
	22. 19 19								
	23. 7 33	17·57	2·22	0·11	116	0·11	274	0·03	285

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
1842. July	23.	19 31	17.57	2.33	0.12	346	0.20	240	0.08	141
	24.	7 43	17.52	2.15	0.06	85	0.14	261	0.03	157
	24.	20 10								
	25.	8 27	17.37	1.98	0.04	201	0.13	256	0.02	144
	25.	20 49								
	26.	8 56	17.22	1.70	0.01	61	0.10	252	0.02	135
	26.	21 15								
	27.	9 35	17.38	1.39	0.02	70	0.10	246	0.04	239
	27.	21 57								
	28.	10 10	17.06	1.17	0.07	31	0.10	256	0.03	126
	28.	22 49								
29.	11 16	17.18	0.93	0.10	64	0.05	310	0.01	183	
29.	23 43									
30.	12 29	17.40	0.80	0.15	78	0.03	323	0.02	330	
31.	1 15									
31.	14 11	17.43	0.94	0.17	173	0.06	329	0.00	17	
August	1.									3 6
	1.	15 41	17.84	1.30	0.10	201	0.09	305	0.03	241
	2.	4 16								
	2.	16 20	17.34	1.39	0.20	260	0.08	222	0.04	173
	3.	4 56								
	3.	17 8	17.77	1.79	0.16	273	0.11	254	0.04	184
	4.	5 31								
	4.	17 42	18.16	2.28	0.15	267	0.17	242	0.05	242
	5.	6 2								
	5.	18 11	18.00	2.72	0.08	201	0.16	239	0.03	129
	6.	6 32								
	6.	18 47	18.19	3.17	0.07	179	0.20	245	0.03	322
	7.	7 2								
	7.	19 19	18.06	3.48	0.23	144	0.13	248	0.03	307
	8.	7 44								
	8.	19 59	18.03	3.32	0.20	161	0.10	250	0.02	321
	9.	8 21								
	9.	20 45	17.91	2.91	0.20	155	0.14	247	0.03	226
	10.	9 6								
	10.	21 27	17.57	2.37	0.21	144	0.11	266	0.02	310
	11.	9 48								
	11.	22 19	17.83	1.62	0.19	154	0.12	257	0.02	39
	12.	10 54								
	13.	0 1	17.58	1.27	0.11	319	0.06	302	0.04	163
	13.	12 55								
	14.	1 49	17.39	1.00	0.07	239	0.04	7	0.03	208
	14.	14 35								
	15.	3 21	17.23	1.06	0.10	244	0.01	82	0.04	181
	15.	15 55								
	16.	4 28	17.21	1.30	0.06	261	0.06	330	0.03	215
	16.	16 50								
	17.	5 15	17.68	1.46	0.06	77	0.09	300	0.03	239
	17.	17 33								
	18.	5 49	17.62	1.67	0.02	67	0.09	293	0.04	56
	18.	18 2								
	19.	6 19	17.88	2.03	0.04	173	0.11	277	0.03	187
	19.	18 13								
	20.	6 25	18.16	2.16	0.09	125	0.18	226	0.06	305
	20.	18 45								
	21.	7 0	17.60	2.28	0.04	140	0.12	251	0.04	199
	21.	19 17								
	22.	7 32	17.36	2.42	0.06	129	0.11	286	0.02	215
	22.	19 43								
	23.	7 57	17.89	2.29	0.04	288	0.11	262	0.03	225

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
		ft.	ft.	ft.	°	ft.	°	ft.	°
1842.	h m								
July	23. 20 46	17.92	2.95	0.04	231	0.17	197	0.03	230
	24. 8 58								
	24. 21 8	17.96	2.88	0.07	101	0.13	197	0.01	5
	25. 9 25								
	25. 21 43	17.79	2.82	0.02	342	0.10	166	0.03	75
	26. 9 56	17.55	2.62	0.15	316	0.08	181	0.00	218
	26. 22 9								
	27. 10 28	17.60	2.24	0.07	266	0.09	201	0.02	255
	27. 22 47								
	28. 11 4	17.56	1.92	0.08	307	0.08	201	0.01	65
	28. 23 21								
	29. 11 44	17.50	1.62	0.15	9	0.14	169	0.04	179
	30. 0 6								
	30. 12 42	17.69	1.30	0.20	20	0.08	187	0.05	197
	31. 1 17								
	31. 14 15	17.97	1.42	0.33	119	0.03	222	0.03	293
August	1. 3 13								
	1. 15 30	17.98	1.67	0.25	207	0.13	156	0.05	90
	2. 3 56								
	2. 16 30	17.79	1.92	0.19	218	0.06	163	0.03	289
	3. 5 6								
	3. 17 39	18.22	2.52	0.14	255	0.13	188	0.04	9
	4. 6 2								
	4. 18 43	18.39	3.07	0.14	242	0.18	202	0.03	26
	5. 7 3								
	5. 19 15	18.43	3.67	0.12	187	0.11	208	0.02	104
	6. 7 36								
	6. 19 53	18.59	4.04	0.10	323	0.12	205	0.04	121
	7. 8 8								
	7. 20 39	18.60	4.21	0.12	144	0.17	201	0.11	337
	8. 9 4								
	8. 21 33	18.42	4.16	0.22	224	0.22	202	0.04	342
	9. 9 55								
	9. 22 17	18.23	3.80	0.16	248	0.19	227	0.13	211
	10. 10 35	17.79	3.32	0.10	257	0.17	215	0.02	356
	10. 22 52								
	11. 11 18	17.76	2.65	0.15	271	0.07	193	0.07	155
	11. 23 44								
	12. 12 7	17.62	1.95	0.12	305	0.04	201	0.03	131
	13. 0 29								
	13. 13 15	17.52	1.54	0.05	205	0.03	285	0.02	142
	14. 2 1								
	14. 14 50	17.60	1.63	0.10	146	0.03	131	0.03	155
	15. 3 38								
	15. 16 26	17.77	1.78	0.12	106	0.11	226	0.03	212
	16. 5 2								
	16. 17 28	18.09	2.08	0.05	149	0.05	235	0.01	341
	17. 5 53								
	17. 18 13	18.07	2.40	0.08	188	0.09	183	0.05	216
	18. 6 29								
	18. 18 52	18.24	2.72	0.02	224	0.15	228	0.04	189
	19. 7 9								
	19. 19 14	18.33	3.09	0.09	261	0.14	203	0.07	70
	20. 7 26								
	20. 19 50	17.93	3.08	0.02	281	0.11	188	0.05	345
	21. 8 5								
	21. 20 19	17.66	3.18	0.05	47	0.16	176	0.05	99
	22. 8 34								
	22. 20 40	18.10	3.10	0.09	149	0.24	178	0.04	110

Height of the Water in each individual tide at Donaghadee, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h	m	ft.	ft.	ft.		ft.		ft.	
June	22.	23 23	18.22	4.78	0.14	72°	0.07	161°	0.01	221°
	23.	11 45								
	24.	0 7	18.70	4.90	0.06	325	0.03	221	0.02	61
	24.	12 25								
	25.	0 43	18.78	4.86	0.06	16	0.06	245	0.03	303
	25.	13 2								
	26.	1 21	18.34	4.76	0.06	356	0.09	240	0.07	31
	26.	13 42								
	27.	2 2	17.56	4.51	0.09	246	0.04	251	0.05	65
	27.	14 20								
	28.	2 37	18.01	4.29	0.01	260	0.04	211	0.03	322
	28.	14 59								
	29.	3 20	17.81	3.97	0.08	200	0.04	208	0.06	31
	29.	15 40								
	30.	4 4	17.69	3.77	0.04	238	0.06	241	0.03	94
	30.	16 25								
July	1.	4 50	17.85	3.56	0.08	264	0.03	201	0.01	218
	1.	17 25								
	2.	5 48	17.93	3.30	0.07	308	0.05	222	0.01	251
	2.	18 22								
	3.	6 55	18.00	3.36	0.09	12	0.04	215	0.01	240
	3.	19 31								
	4.	7 58	18.70	3.58	0.10	6	0.04	263	0.03	349
	4.	20 25								
	5.	8 58	18.45	3.90	0.12	24	0.05	277	0.03	258
	5.	21 23								
	6.	9 49	17.58	4.40	0.15	42	0.07	269	0.04	126
	6.	22 13								
	7.	10 32	18.01	4.61	0.09	60	0.01	53	0.05	208
	7.	22 52								
	8.	11 14	18.21	5.12	0.08	40	0.04	172	0.02	297
	8.	23 35								
	9.	11 59	18.00	5.36	0.07	66	0.04	162	0.04	43
	10.	0 22								
	10.	12 45	18.18	5.54	0.05	101	0.10	212	0.03	271
	11.	1 7								
	11.	13 31	18.37	5.71	0.11	17	0.06	199	0.06	247
	12.	1 55								
	12.	14 21	17.93	5.49	0.15	66	0.05	193	0.06	331
	13.	2 47								
	13.	15 13	18.16	5.22	0.09	34	0.07	175	0.03	317
	14.	3 38								
	14.	16 7	17.59	5.06	0.07	182	0.05	236	0.04	57
	15.	4 35								
	15.	17 10	17.28	4.60	0.06	204	0.03	216	0.02	9
	16.	5 44								
	16.	18 21	17.69	4.15	0.07	290	0.05	241	0.02	49
	17.	6 48								
	17.	19 29	17.96	3.88	0.11	337	0.11	230	0.06	56
	18.	8 13								
	18.	20 39	17.93	3.85	0.11	13	0.02	312	0.03	198
	19.	9 5								
	19.	21 35	17.74	4.01	0.08	71	0.01	257	0.01	236
	20.	9 59								
	20.	22 23	17.66	4.30	0.07	333	0.01	41	0.03	11
	21.	10 42								
	21.	23 1	17.64	4.53	0.07	334	0.01	250	0.04	41
	22.	11 19								
	22.	23 37	17.58	4.80	0.04	328	0.01	132	0.02	159
	23.	11 55								
	23.	23 37	17.50	4.90	0.07	328	0.03	123	0.00	31

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.		A_0	C_1	C_2	c_2	C_3	c_3	C_4	c_4
1842.	h m	ft.	ft.	ft.	11°	ft.	163°	ft.	136°
July	24. 0 12	17.54	4.91	0.03		0.02		0.03	
	24. 12 29								
	25. 0 46	17.55	4.87	0.03	104	0.05	140	0.01	151
	25. 13 2								
	26. 1 18	17.57	4.71	0.06	110	0.01	214	0.03	61
	26. 13 36								
	27. 1 54	17.43	4.56	0.02	157	0.02	318	0.04	314
	27. 14 12								
	28. 2 31	17.58	4.31	0.08	118	0.02	212	0.02	74
	28. 14 49								
	29. 3 9	17.28	4.08	0.09	206	0.08	233	0.05	71
	29. 15 26								
	30. 3 46	17.39	3.72	0.08	251	0.08	246	0.04	112
	30. 16 18								
	31. 4 51	17.39	3.40	0.09	261	0.04	251	0.01	111
	31. 17 22								
August	1. 6 1	17.55	3.24	0.11	314	0.02	179	0.03	124
	1. 18 45								
	2. 7 17	17.97	3.26	0.12	9	0.03	211	0.03	147
	2. 19 58								
	3. 8 37	17.78	3.69	0.12	60	0.08	69	0.02	325
	3. 21 8	17.98	4.11	0.03	122	0.04	164	0.00	179
	4. 9 37								
	4. 22 7	18.24	4.81	0.06	6	0.04	158	0.01	117
	5. 10 23								
	5. 22 39	17.99	5.28	0.09	99	0.02	162	0.07	321
	6. 11 0								
	6. 23 22	18.18	5.72	0.13	52	0.05	174	0.02	104
	7. 11 44								
	8. 0 5	17.88	5.98	0.10	68	0.06	152	0.05	319
	8. 12 29								
	9. 0 52	18.02	6.01	0.15	67	0.07	174	0.05	3
	9. 13 14								
	10. 1 36	18.03	5.89	0.11	51	0.06	166	0.01	135
	10. 14 1								
	11. 2 25	17.93	5.46	0.14	92	0.05	196	0.06	9
	11. 14 47								
	12. 3 9	17.93	5.20	0.11	221	0.03	237	0.03	28
	12. 15 40								
	13. 4 8	17.76	4.50	0.16	224	0.03	223	0.02	82
	13. 16 42								
	14. 5 17	17.50	3.92	0.06	234	0.03	232	0.03	108
	14. 18 0								
	15. 6 45	17.46	3.45	0.08	277	0.01	99	0.02	41
	15. 19 22								
	16. 7 59	17.35	3.42	0.05	6	0.03	286	0.03	192
	16. 20 33								
	17. 9 3	17.81	3.77	0.06	4	0.04	218	0.01	138
	17. 21 29	17.74	4.02	0.05	347	0.03	164	0.01	43
	18. 9 51								
	18. 22 12	17.98	4.51	0.07	344	0.06	168	0.05	71
	19. 10 31								
	19. 22 49	18.12	4.54	0.09	25	0.09	81	0.06	36
	20. 11 5								
	20. 23 21	17.65	4.90	0.01	141	0.05	202	0.03	341
	21. 11 37								
	21. 23 52	17.25	4.95	0.04	30	0.06	179	0.04	182
	22. 12 6								
	23. 0 20	17.73	5.02	0.06	129	0.07	164	0.04	164

Height of the Water in each individual tide at Ardglass, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.		ft.		ft.	
June	22. 23 30	18.35	6.30	0.20	87°	0.12	175°	0.04	211°
	23. 11 45								
	24. 0 0	18.60	6.35	0.17	73	0.12	231	0.09	188

	26. 1 13	18.20	6.11	0.15	110	0.05	216	0.03	105
	26. 13 34								
	27. 1 55	17.62	5.76	0.10	96	0.07	146	0.04	28
	27. 14 14								
	28. 2 33	17.83	5.40	0.10	99	0.06	188	0.02	61
	28. 14 54								
	29. 3 15	17.76	4.99	0.14	125	0.05	170	0.01	312
	29. 15 36								
	30. 4 0	17.59	4.71	0.09	131	0.11	217	0.02	176
	30. 16 26								
July	1. 4 51	17.71	4.38	0.09	173	0.06	196	0.02	195
	1. 17 22								
	2. 5 45	17.77	4.02	0.06	233	0.07	238	0.01	267
	2. 18 22								
	3. 6 55	17.81	4.06	0.01	42	0.04	240	0.02	224
	3. 19 33								
	4. 8 0	18.49	4.33	0.07	345	0.10	244	0.02	43
	4. 20 27								
	5. 9 0	18.22	4.81	0.24	71	0.15	301	0.14	176
	5. 21 22								
	6. 9 48	17.41	5.60	0.12	77	0.06	222	0.05	146

	7. 22 56	18.23	6.68	0.12	138	0.11	5	0.08	190
	8. 11 16								
	8. 23 35	17.76	7.08	0.19	90	0.05	47	0.00	208
	9. 11 56								
	10. 0 17	17.90	7.24	0.18	93	0.09	157	0.09	35
	10. 12 41								
	11. 1 5	18.29	7.62	0.15	64	0.16	196	0.16	145
	11. 13 30								
	12. 1 54	17.75	7.30	0.22	100	0.01	106	0.01	270
	12. 14 18								
	13. 2 43	17.87	6.78	0.15	69	0.08	137	0.05	109
	13. 15 11								
	14. 3 36	17.54	6.53	0.16	116	0.04	128	0.05	215
	14. 16 9								
	15. 4 37	17.24	5.81	0.15	121	0.02	234	0.03	254
	15. 17 13								
	16. 5 47	17.52	5.10	0.05	138	0.05	246	0.02	81

	17. 19 40								
	18. 8 24	17.77	4.88	0.07	356	0.08	250	0.06	179
	18. 20 52								
	19. 9 18	17.74	5.26	0.07	53	0.04	232	0.06	171
	19. 21 47								
	20. 10 11	17.64	5.50	0.08	60	0.04	296	0.06	124
	20. 22 35								
	21. 10 54	17.60	5.90	0.07	118	0.08	189	0.03	116
	21. 23 12								
	22. 11 29	17.54	6.18	0.11	89	0.06	187	0.04	254
	22. 23 46								
	23. 12 3	17.46	6.39	0.11	132	0.06	143	0.01	112

pressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
		ft.	ft.	ft.		ft.		ft.	
1842.	h m								
July	24. 0 20	17.48	6.40	0.10	140	0.08	144	0.03	199
	24. 12 38								
	25. 0 55	17.48	6.34	0.16	99	0.03	174	0.03	26
	25. 13 9								
	26. 1 24	17.52	6.18	0.15	118	0.05	144	0.07	196
	26. 13 41								
	27. 1 58	17.40	5.85	0.21	118	0.07	148	0.02	200
	27. 14 15								
	28. 2 33	17.47	5.48	0.20	117	0.03	123	0.03	157
	28. 14 53	17.29	5.07	0.16	129	0.09	187	0.06	213
	29. 3 13								
	29. 15 33	17.35	4.67	0.12	148	0.04	221	0.03	155
	30. 3 53								
	30. 16 21	17.34	4.15	0.12	156	0.01	257	0.05	170
	31. 4 54								
	31. 17 28	17.38	3.88	0.07	198	0.02	255	0.01	77
August	1. 6 7								
	1. 18 48	17.71	3.98	0.09	351	0.05	234	0.03	143
	2. 7 20								
	2. 20 2	17.58	4.63	0.09	63	0.03	55	0.02	197
	3. 8 41								
	3. 21 8	17.85	5.25	0.13	105	0.04	168	0.05	209
	4. 9 32								
	4. 21 55	18.03	6.21	0.12	137	0.08	132	0.04	165
	5. 10 18								
	5. 22 42	17.91	6.99	0.24	99	0.02	140	0.06	146
	6. 11 1								
	6. 23 20	18.05	7.60	0.26	78	0.03	124	0.09	127
	7. 11 42								
	8. 0 3	17.83	8.11	0.27	89	0.12	109	0.04	327
	8. 12 25								
	9. 0 47	17.88	8.12	0.25	95	0.07	138	0.09	79
	9. 13 9								
	10. 1 32	17.85	7.80	0.20	87	0.07	139	0.08	57
	10. 13 56								
	11. 2 20	17.81	7.13	0.32	91	0.04	88	0.03	68
	11. 14 51	17.88	6.64	0.18	133	0.08	173	0.07	73
	12. 3 13								
	12. 15 46	17.70	5.62	0.14	132	0.06	193	0.10	127
	13. 4 14								
	13. 16 51	17.42	4.81	0.10	139	0.03	186	0.01	158
	14. 5 26								
	14. 18 7	17.36	4.22	0.04	172	0.02	313	0.01	253
	15. 6 52								
	15. 19 38	17.30	4.29	0.06	95	0.02	84	0.02	182
	16. 8 15								
	16. 20 46	17.71	4.75	0.13	88	0.03	253	0.04	135
	17. 9 16								
	17. 21 40	17.66	5.20	0.08	140	0.02	174	0.02	149
	18. 10 1								
	18. 22 21	17.91	5.80	0.13	76	0.16	211	0.10	99
	19. 10 36								
	19. 22 51	17.99	6.00	0.24	62	0.05	204	0.03	92
	20. 11 5								
	20. 23 18	17.57	6.45	0.27	146	0.10	236	0.09	340
	21. 11 37								
	21. 23 55	17.15	6.53	0.20	117	0.05	166	0.03	27
	22. 12 7								
	23. 0 19	17.62	6.57	0.14	111	0.06	111	0.05	210

Height of the Water in each individual tide at Clogher Head, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°
.....
.....
June	24. 0 6	17·96	6·22	0·22	126	0·07	161	0·05	200
	24. 12 24								
	25. 0 42	17·93	6·16	0·25	113	0·06	166	0·03	307
	25. 12 59								
	26. 1 16	17·48	5·99	0·23	124	0·04	202	0·05	167
	26. 13 37								
	27. 1 57	17·15	5·58	0·27	116	0·07	145	0·02	162
	27. 14 17								
	28. 2 36	17·35	5·30	0·16	119	0·05	120	0·04	177
	28. 14 57								
	29. 3 18	17·29	4·79	0·21	123	0·05	153	0·03	233
	29. 15 39								
	30. 4 3	17·13	4·50	0·19	137	0·02	85	0·08	178
	30. 16 27								
July	1. 4 52	17·26	4·18	0·13	149	0·10	190	0·03	116
	1. 17 27								
	2. 5 50	17·21	3·83	0·18	178	0·10	281	0·05	156
	2. 18 29								
	3. 7 2	17·34	3·99	0·08	175	0·08	212	0·03	68
	3. 19 32								
	4. 7 59	17·88	4·28	0·06	336	0·10	227	0·07	205
	4. 20 31								
	5. 9 4	17·52	4·90	0·10	141	0·06	52	0·04	126
	5. 21 28								
	6. 9 53	17·01	5·28	0·20	117	0·03	184	0·03	207
	6. 22 17								
	7. 10 37	17·40	6·00	0·21	113	0·11	145	0·10	106
	7. 22 57								
	8. 11 21	17·61	6·61	0·24	119	0·08	118	0·04	113
	8. 23 44								
	9. 12 7	17·53	7·11	0·41	111	0·05	101	0·05	325
	10. 0 29								
	10. 12 54	17·54	7·36	0·34	111	0·05	75	0·02	85
	11. 1 19								
	11. 13 43	17·95	7·50	0·29	123	0·08	203	0·10	253
	12. 2 6								
	12. 14 31	17·29	7·27	0·40	124	0·06	202	0·05	274
	13. 2 55								
	13. 15 15	17·34	6·71	0·25	103	0·04	170	0·08	194
	14. 3 40								
	14. 16 24	16·72	6·16	0·07	291	0·23	124	0·07	209
	15. 4 52								
	15. 17 28	16·84	5·68	0·24	133	0·02	185	0·04	254
	16. 6 2								
	16. 18 32	17·14	5·10	0·14	113	0·09	226	0·03	45
	17. 7 0								
	17. 19 44	17·10	4·95	0·13	205	0·14	204	0·08	329
	18. 8 28								
	18. 21 6	17·30	4·60	0·11	288	0·16	159	0·11	336
	19. 9 32								
	19. 21 59	17·14	5·20	0·11	195	0·07	252	0·07	187
	20. 10 23								
	20. 22 46	17·26	5·20	0·11	339	0·13	170	0·10	274
	21. 11 3								
	21. 23 19	17·15	5·61	0·18	121	0·09	306	0·05	122
	22. 11 37								
	22. 23 54	17·10	6·04	0·22	125	0·06	146	0·04	291
	23. 12 13								
		17·11	6·02	0·19	76	0·25	165	0·14	310

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin (2p + c_2) + C_3 \sin (3p + c_3) + C_4 \sin (4p + c_4)$.

Analysed time of high water, corresponding to $p = 90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
1842. July	23.	18 22	15.72	5.39	0.18	13	0.14	96	0.08	141
	24.	6 38								
	24.	18 55	15.82	5.35	0.13	17	0.19	76	0.05	244
	25.	7 15								
	25.	19 30	15.77	5.21	0.19	31	0.21	47	0.08	316
	26.	7 47								
	26.	20 2	15.65	4.90	0.12	339	0.15	95	0.05	284
	27.	8 18								
	27.	20 33	15.57	4.62	0.14	8	0.18	91	0.03	213
	28.	8 49								
	28.	21 5	15.54	4.22	0.20	322	0.21	93	0.02	254
29.	9 22									
29.	21 39	15.66	3.86	0.10	352	0.13	84	0.05	191	
30.	10 4									
30.	22 28	15.51	3.33	0.18	328	0.09	116	0.04	96	
31.	11 2									
31.	23 36	15.64	2.92	0.11	3	0.01	125	0.02	285	
August	1. 12 22									
2.	1 7	15.89	2.90	0.13	7	0.05	196	0.03	147	
2.	13 50									
3.	2 33	15.72	3.55	0.08	107	0.04	143	0.06	269	
3.	15 7									
4.	3 39	15.69	4.29	0.08	50	0.11	109	0.04	52	
4.	16 13									
5.	4 33	15.87	5.00	0.12	23	0.20	105	0.08	347	
5.	16 57									
6.	5 21	15.91	5.90	0.17	10	0.12	95	0.08	176	
6.	17 49									
7.	6 9	15.93	6.40	0.14	22	0.16	100	0.01	133	
7.	18 34									
8.	6 57	15.86	6.60	0.10	343	0.20	68	0.10	40	
8.	19 20									
9.	7 41	15.99	6.68	0.09	6	0.23	80	0.11	5	
9.	20 4									
10.	8 24	16.00	6.51	0.10	346	0.21	108	0.01	37	
10.	20 44									
11.	9 4	15.68	5.92	0.14	358	0.27	107	0.04	29	
11.	21 24									
12.	9 47	15.72	5.33	0.22	5	0.22	93	0.02	140	
12.	22 10									
13.	10 37	15.53	4.46	0.02	6	0.23	101	0.02	322	
13.	23 3									
14.	11 42	15.39	3.64	0.18	329	0.13	128	0.01	50	
15.	0 21									
15.	13 6	15.56	3.06	0.14	10	0.08	137	0.01	84	
16.	1 51									
16.	14 30	15.57	3.23	0.09	14	0.07	167	0.02	131	
17.	3 9									
17.	15 30	16.00	3.80	0.10	17	0.08	108	0.03	158	
18.	4 0									
18.	16 22	16.03	4.31	0.22	0	0.12	127	0.05	120	
19.	4 43									
19.	17 5	16.15	4.85	0.06	71	0.23	57	0.10	235	
20.	5 19									
20.	17 40	15.94	5.03	0.10	353	0.12	120	0.04	219	
21.	5 56									
21.	18 16	15.77	5.38	0.08	347	0.15	95	0.07	203	
22.	6 28									
22.	18 41	15.46	5.60	0.14	356	0.18	84	0.03	257	
23.	6 59									
23.	16 00	16.00	5.59	0.06	26	0.23	80	0.03	326	

Height of the Water in each individual tide at New Ross, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.		A_0	C_1	C_2	c_2	C_3	c_3	C_4	c_4	
1842.	h m	ft.	ft.	ft.	°	ft.	°	ft.	°	
June	22.	18 14	17.32	5.88	0.05	350	0.31	93	0.01	259
	23.	6 36								
	23.	18 55	17.47	5.85	0.40	350	0.34	92	0.17	140
	24.	7 23								
	24.	19 44	17.47	5.65	0.29	340	0.40	108	0.04	107
	25.	7 54								
	25.	20 13	16.80	5.61	0.61	342	0.22	100	0.05	104
	26.	8 27								
	26.	20 51	16.57	5.43	0.43	334	0.32	101	0.02	58
	27.	9 6								
	27.	21 21	16.61	5.18	0.37	334	0.27	91	0.01	322
	28.	9 38								
	28.	21 54	16.57	4.83	0.32	329	0.23	87	0.03	300
	29.	10 14								
	29.	22 33	16.53	4.50	0.37	343	0.25	89	0.11	114
	30.	10 52								
	30.	23 10	16.55	4.25	0.27	328	0.15	85	0.03	10
July	1.	11 39								
	2.	0 7	16.50	3.75	0.30	331	0.17	89	0.02	69
	2.	12 42								
	3.	1 18	16.96	3.69	0.26	354	0.19	91	0.01	222
	3.	13 54								
	4.	2 29	17.56	3.96	0.13	327	0.13	104	0.05	25
	4.	14 54								
	5.	3 35	17.19	4.34	0.25	350	0.17	99	0.03	158
	5.	16 14								
	6.	4 45	16.30	5.00	0.33	355	0.23	107	0.05	137
	6.	17 14								
	7.	5 41	16.94	5.48	0.44	355	0.28	99	0.07	109
	7.	18 3								
	8.	6 21	17.11	5.88	0.51	341	0.39	97	0.03	106
	8.	18 58								
	9.	7 18	16.86	6.22	0.52	339	0.36	109	0.04	116
	9.	19 49	17.03	6.43	0.53	346	0.38	104	0.07	131
	10.	8 8								
	10.	20 28	17.74	6.39	0.51	343	0.37	93	0.06	108
	11.	8 54								
	11.	21 19	17.04	6.34	0.66	344	0.35	99	0.06	56
	12.	9 43								
	12.	22 7	16.97	6.13	0.56	348	0.38	92	0.09	97
	13.	10 29								
	13.	22 48	16.39	5.67	0.49	341	0.30	95	0.03	104
	14.	11 12								
	14.	23 36	16.23	5.18	0.46	336	0.24	100	0.02	63
	15.	12 2								
	16.	0 27	16.81	4.67	0.39	343	0.26	96	0.01	327
	16.	12 55								
	17.	1 23	16.98	4.24	0.31	342	0.21	123	0.02	312
	17.	14 6								
	18.	2 49	16.92	4.25	0.30	358	0.18	103	0.01	246
	18.	15 27								
	19.	4 0	16.79	4.61	0.42	347	0.16	100	0.00	270
	19.	16 37								
	20.	5 4	16.79	5.08	0.40	352	0.21	104	0.02	352
	20.	17 33								
	21.	5 51	16.64	5.42	0.46	347	0.25	101	0.01	116
	21.	18 14								
	22.	6 30	16.69	5.75	0.52	347	0.29	91	0.03	68
	22.	18 53								
	23.	7 10	16.64	5.87	0.50	351	0.34	101	0.06	116

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin (2p + c_2) + C_3 \sin (3p + c_3) + C_4 \sin (4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.		A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .									
		ft.	ft.	ft.	°	ft.	°	ft.	°									
1842. July	23.	19 27	16.59	5.93	0.51	351	0.31	93	0.10	109								
	24.	7 43																
	24.	19 58																
	25.	8 18																
	25.	20 31																
	26.	8 46																
	26.	21 0																
	27.	9 15																
	27.	21 31																
	28.	9 45																
	28.	21 58																
29.	10 15																	
29.	22 32																	
30.	10 51																	
30.	23 10																	
31.	11 41																	
August	1.	0 12	16.35	3.58	0.32	329	0.13	90	0.01	242								
	1.	12 54	16.66	3.47	0.27	14	0.15	104	0.06	159								
	2.	1 36																
	2.	14 23																
	3.	3 10									16.40	4.17	0.20	6	0.13	94	0.05	227
	3.	15 42									16.50	4.90	0.30	352	0.21	97	0.02	70
	4.	4 21																
	4.	16 54									16.82	5.61	0.43	341	0.35	85	0.02	314
	5.	5 14									16.78	6.30	0.59	348	0.35	101	0.07	101
	5.	17 57																
	6.	6 21									17.05	6.76	0.58	351	0.39	100	0.07	90
	6.	18 47																
	7.	7 7									17.15	6.89	0.59	353	0.41	91	0.05	79
	7.	19 34																
	8.	7 57									17.13	6.95	0.68	348	0.42	98	0.12	92
	8.	20 27																
	9.	8 43									17.14	6.81	0.63	349	0.49	93	0.16	113
	9.	20 59																
	10.	9 21									16.77	6.31	0.59	342	0.39	106	0.09	85
	10.	21 42																
	11.	9 58									16.88	5.86	0.49	340	0.39	101	0.06	132
	11.	22 15																
	12.	10 36									16.33	5.09	0.41	336	0.26	99	0.03	66
12.	22 57																	
13.	11 22	16.10	4.26	0.37	323	0.17	103	0.05	34									
13.	23 46																	
14.	12 22	16.26	3.70	0.25	343	0.17	105	0.05	301									
15.	0 58																	
15.	13 44	16.28	3.82	0.24	347	0.14	100	0.01	122									
16.	2 30																	
16.	15 14	16.52	4.20	0.27	343	0.13	122	0.04	43									
17.	3 44																	
17.	16 15	16.63	4.91	0.35	358	0.23	80	0.09	141									
18.	4 47																	
18.	17 11	16.87	5.39	0.38	348	0.28	95	0.05	164									
19.	5 32																	
19.	17 57	16.83	5.52	0.39	339	0.26	108	0.04	185									
20.	6 11																	
20.	18 35	16.59	5.96	0.56	347	0.25	93	0.08	69									
21.	6 51																	
21.	19 12	16.18	6.03	0.50	344	0.39	97	0.08	110									
22.	7 24																	
22.	19 33	16.84	6.09	0.48	352	0.34	92	0.01	144									
23.	7 51																	

Height of the Water in each individual tide at Passage West, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
.....
.....
.....
.....
June	24.	19 6	16.13	5.37	0.36	279	0.06	104	0.03	92
	25.	7 16
.....
.....
	26.	20 6	15.52	5.02	0.31	290	0.12	80	0.03	68
	27.	8 25
	27.	20 43	15.55	4.64	0.32	289	0.11	60	0.03	122
	28.	9 3
	28.	21 23	15.53	4.30	0.31	295	0.16	70	0.01	58
	29.	9 40
	29.	22 4	15.62	3.91	0.33	305	0.06	67	0.02	94
	30.	10 34
	30.	23 20	15.69	3.73	0.23	301	0.07	116	0.01	6
July	1.	11 33
	1.	23 46	15.64	3.46	0.26	316	0.10	97	0.03	129
	2.	12 23
	3.	0 59	16.11	3.38	0.25	358	0.08	158	0.08	298
	3.	13 32
	4.	2 4	16.40	3.73	0.20	331	0.03	157	0.03	109
	4.	14 34
	5.	3 4	15.73	4.20	0.14	312	0.03	123	0.03	83
	5.	15 36
	6.	4 6	15.34	4.65	0.19	300	0.07	101	0.03	74
	6.	16 32
	7.	4 59	15.92	5.11	0.21	331	0.07	102	0.07	69
	7.	17 22
	8.	5 40	15.76	5.70	0.28	310	0.09	92	0.01	88
	8.	18 8
	9.	6 28	15.63	6.03	0.36	299	0.10	131	0.05	31
	9.	18 52
	10.	7 16	16.01	6.25	0.25	284	0.13	38	0.02	353
	10.	19 36
	11.	8 0	16.38	6.38	0.27	308	0.06	102	0.09	103
	11.	20 28
	12.	8 51	15.72	6.21	0.32	307	0.12	98	0.07	89
	12.	21 15	15.52	5.96	0.36	309	0.20	80	0.10	64
	13.	9 39
	13.	22 3	15.32	5.36	0.30	309	0.17	84	0.06	64
	14.	10 31
	14.	22 58	15.39	4.86	0.36	313	0.14	99	0.04	106
	15.	11 28
	15.	23 58	16.05	4.31	0.34	327	0.11	108	0.02	75
	16.	12 32
	17.	1 6	16.13	3.91	0.28	307	0.06	118	0.04	71
	17.	13 45
	18.	2 23	16.09	3.99	0.22	331	0.05	130	0.01	38
	18.	14 57
	19.	3 31	15.96	4.41	0.26	315	0.10	75	0.04	132
	19.	15 59
	20.	4 26	16.01	4.69	0.31	332	0.06	105	0.03	52
	20.	16 51
	21.	5 9	15.71	5.08	0.38	315	0.10	101	0.02	71
	21.	17 31
	22.	5 47	15.79	5.38	0.35	307	0.07	102	0.05	91
	22.	18 6
	23.	6 23	15.68	5.53	0.33	301	0.09	72	0.01	36

expressed by the formula $A_0 + C_1 \sin p + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$.

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .	
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°	
July	23.	18	39	15.73	5.61	0.33	291	0.07	60	0.04	173
	24.	6	55	15.75	5.58	0.30	293	0.14	48	0.07	94
	24.	19	7								
	25.	7	27	15.66	5.34	0.28	288	0.08	67	0.08	103
	25.	19	39								
	26.	7	56	15.60	5.08	0.34	287	0.13	82	0.02	238
	26.	20	12								
	27.	8	28	15.41	4.73	0.29	291	0.12	65	0.04	79
	27.	20	44								
	28.	8	59	15.54	4.48	0.26	291	0.17	51	0.04	58
	28.	21	18								
	29.	9	37	15.57	3.98	0.28	303	0.14	73	0.00	28
	29.	21	55								
	30.	10	21	15.45	3.53	0.29	302	0.08	104	0.02	325
30.	22	46									
31.	11	19	15.65	3.25	0.25	329	0.07	108	0.03	343	
31.	23	51									
August	1.	12	33	15.86	3.30	0.14	357	0.07	173	0.00	262
	2.	1	15	15.65	3.91	0.12	312	0.04	55	0.01	213
	3.	2	40								
	3.	15	8	15.52	4.59	0.23	316	0.10	150	0.03	70
	4.	3	40								
	4.	16	13	15.67	5.29	0.29	316	0.10	135	0.03	91
	5.	4	33								
	5.	17	1	15.79	6.12	0.25	298	0.06	111	0.08	86
	6.	5	25								
	6.	17	50	15.79	6.62	0.21	299	0.09	106	0.09	88
	7.	6	10								
	7.	18	34	15.78	6.89	0.30	294	0.08	110	0.06	37
	8.	6	57								
	8.	19	19	15.95	6.93	0.30	275	0.04	140	0.09	34
	9.	7	40								
	9.	20	2	15.89	6.63	0.29	289	0.11	75	0.04	38
	10.	8	26								
	10.	20	46	15.64	6.20	0.36	297	0.15	79	0.06	46
	11.	9	7	15.68	5.60	0.28	298	0.17	77	0.11	100
	11.	21	28								
	12.	9	54	15.43	4.72	0.35	312	0.15	83	0.04	131
	12.	22	20								
	13.	10	51	15.31	3.90	0.37	314	0.11	91	0.02	25
13.	23	21									
14.	12	1	15.46	3.50	0.26	343	0.10	134	0.07	30	
15.	0	40									
15.	13	23	15.58	3.60	0.19	329	0.05	158	0.05	24	
16.	2	6									
16.	14	40	15.84	4.14	0.22	317	0.05	90	0.03	33	
17.	3	13									
17.	15	44	15.99	4.47	0.23	339	0.08	50	0.06	152	
18.	4	14									
18.	16	27	16.01	5.02	0.25	312	0.11	73	0.04	99	
19.	4	48									
19.	17	10	15.80	5.30	0.34	311	0.08	96	0.09	69	
20.	5	24									
20.	17	40	15.65	5.60	0.27	296	0.05	44	0.08	73	
21.	5	56									
21.	18	14	15.31	5.80	0.33	291	0.07	64	0.08	51	
22.	6	26									
22.	18	40	15.83	5.78	0.28	295	0.10	63	0.09	54	
23.	6	58									

Height of the Water in each individual tide at Castle Townsend, excluding diurnal tide,

Analysed time of high water, corresponding to $p=90^\circ$.			A_0 .	C_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h	m	ft.	ft.	ft.	°	ft.	°	ft.	°
June 22.	17	1	15·86	4·31	0·15	283	0·07	208	0·02	170
23.	5	20	15·77	4·49	0·17	276	0·02	77	0·05	277
23.	17	45								
24.	6	10	15·72	4·21	0·27	266	0·07	259	0·03	13
24.	18	36								
25.	6	46	15·41	4·15	0·21	271	0·03	239	0·03	203
25.	19	7								
26.	7	21	15·21	3·98	0·23	292	0·02	18	0·02	3
26.	19	41								
27.	8	0	15·20	3·65	0·17	277	0·04	243	0·01	309
27.	20	20								
28.	8	40	15·30	3·37	0·16	315	0·06	51	0·13	63
28.	21	0								
29.	9	17	15·43	3·11	0·16	306	0·04	269	0·02	182
29.	21	40								
30.	10	10	15·29	2·83	0·19	317	0·05	315	0·02	147
30.	22	39								
July 1.	11	8	15·44	2·70	0·16	323	0·01	221	0·02	266
1.	23	37								
2.	12	6	15·92	2·80	0·04	259	0·09	263	0·04	338
3.	0	35								
3.	13	12	16·04	3·01	0·15	345	0·14	295	0·02	35
4.	1	48								
4.	14	15	15·42	3·37	0·17	299	0·07	328	0·04	190
5.	2	42								
5.	15	12	15·16	3·85	0·12	312	0·08	259	0·09	31
6.	3	41								
6.	16	2	15·42	4·34	0·17	276	0·06	344	0·01	61
7.	4	24								
.....
8.	17	35	15·27	5·06	0·19	296	0·06	265	0·06	294
9.	5	55								
9.	18	10	15·34	5·06	0·37	325	0·17	203	0·07	34
10.	6	34								
10.	18	59	15·92	5·28	0·18	280	0·04	313	0·06	309
11.	7	23								
11.	19	50	15·35	5·11	0·27	296	0·04	274	0·04	106
12.	8	13								
12.	20	42	15·12	4·77	0·24	269	0·05	264	0·09	256
13.	9	6								
13.	21	32	14·85	4·32	0·26	287	0·06	222	0·10	30
14.	9	59								
.....
15.	23	29	15·77	3·47	0·17	262	0·11	285	0·07	16
16.	12	6	15·65	3·55	0·18	1	0·13	77	0·13	108
17.	0	43								
17.	13	22	15·72	3·31	0·18	295	0·06	284	0·04	154
18.	2	0								
18.	14	33	15·66	3·61	0·19	307	0·03	323	0·05	202
19.	3	5								
19.	15	32	15·66	3·92	0·27	316	0·07	329	0·01	337
20.	3	59								
20.	16	26	15·47	4·16	0·25	318	0·04	52	0·04	137
21.	4	44								
21.	17	2	15·59	4·31	0·17	303	0·08	237	0·03	243
22.	5	18								
22.	17	35	15·41	4·40	0·23	281	0·05	116	0·01	184
23.	5	52								

Section XI.—*Discussion of the height of mean water deduced from the analysis of individual tides ; with reference to difference of station, and to variations of the phase of the moon, and of the declination of the moon.*

The mean heights (A_0) for each station, in the results of last section, were divided into groups corresponding to large tides and small tides, the dividing places being the same as those in section V., page 29, or following by two days the times when the moon's hour-angle from the sun was 3^h , 9^h , 15^h , 21^h . The means for each group were taken, and then the mean of those means of groups belonging to large tides, and the similar mean for small tides. The results are the following, the numbers for Courtown being taken by anticipation from section XVII. :—

Station.	Mean height.		Mean of mean heights.	Excess of mean height in large tides above mean height in small tides.
	Small tides.	Large tides.		
	ft.	ft.	ft.	ft.
Kilbaha	15·58	15·72	15·65	0·14
Kilrush	16·36	16·52	16·44	0·16
Foynes Island	17·06	17·29	17·18	0·23
Limerick	17·38	17·92	17·65	0·54
Casleh Bay	17·90	18·06	17·98	0·16
Galway	17·47	17·65	17·56	0·18
Old Head	17·69	17·90	17·80	0·21
Mullaghmore	18·03	18·23	18·13	0·20
Buncrana	17·33	17·47	17·40	0·14
Port Rush	17·60	17·83	17·72	0·23
Carrowkeel	17·96	18·17	18·07	0·21
Ballycastle	17·40	17·63	17·52	0·23
Glenarm	17·65	17·83	17·74	0·18
Donaghadee	17·76	17·89	17·83	0·13
Ardglass	17·65	17·78	17·72	0·13
Clogher Head	17·20	17·32	17·26	0·12
Kingstown	17·44	17·49	17·47	0·05
Courtown	16·63	16·67	16·65	0·04
Dunmore East	15·80	15·86	15·83	0·06
New Ross	16·58	16·89	16·74	0·31
Passage West	15·71	15·78	15·75	0·07
Castle Townsend	15·45	15·45	15·45	0·00

The first column which deserves attention is the "mean of mean heights." The progress of the numbers from Kilbaha to Limerick, as well as that from Dunmore East to New Ross, show well the change of mean height in ascending a river affected by current as well as by tide. Excluding these river stations, as also Buncrana and Carrowkeel, which partake in some measure of the same character, we have a view of the comparative mean heights of the sea on different parts of the coast of Ireland. And here we have the remarkable result, to which allusion has already been made, that the mean height of the sea round the northern half of the island, as referred to the surface of stagnant water, is considerably greater than that round the southern half of the island. The amount of this difference of height is believed by the officers who directed the levelling operations to be much greater than can be explained by any allowable error in the levelling. The heights on the eastern coast are also, perhaps, a little greater than those on the western coast. I profess myself entirely unable to explain on mechanical principles this result.

In every instance except that of Castle Townsend, the mean height in large tides is greater than that in small tides. Further allusion will be made to this in the examination of the next table. I shall here only remark that I imagine this to be a possible result of the shallowness of the sea, though theory has not yet reached so far.

For the investigation of the effect of the moon's declination, the same process in all respects was used as in Section V., pages 32 and 33, and the following are the results:—

Station.	Mean height with small declination.	Mean height with large declination.	Excess of mean height with large declination above mean height with small declination.	Mean height with decreasing declination.	Mean height with increasing declination.	Excess of mean height with decreasing declination above mean height with increasing declination.
	ft.	ft.	ft.	ft.	ft.	ft.
Kilbaha	15·61	15·66	+0·05	15·68	15·59	+0·09
Kilrush	16·40	16·46	+0·06	16·52	16·34	+0·18
Foynes Island....	17·15	17·18	+0·03	17·24	17·07	+0·17
Limerick.....	17·68	17·57	-0·11	17·81	17·43	+0·38
Casleh Bay.....	17·94	18·00	+0·06	18·03	17·91	+0·12
Galway	17·54	17·58	+0·04	17·62	17·49	+0·13
Old Head	17·71	17·81	+0·10	17·91	17·69	+0·22
Mullaghmore....	18·07	18·15	+0·08	18·22	18·06	+0·16
Buncrana	17·37	17·42	+0·05	17·53	17·31	+0·22
Port Rush	17·68	17·70	+0·02	17·88	17·59	+0·29
Carrowkeel.....	17·97	18·12	+0·15	18·18	17·94	+0·24
Ballycastle.....	17·47	17·59	+0·12	17·64	17·39	+0·25
Glenarm	17·68	17·76	+0·08	17·87	17·66	+0·21
Donaghadee	17·79	17·85	+0·06	17·93	17·77	+0·16
Ardglass.....	17·69	17·73	+0·04	17·82	17·64	+0·18
Clogher Head ..	17·21	17·29	+0·08	17·29	17·20	+0·09
Kingstown	17·41	17·50	+0·09	17·53	17·42	+0·11
Courtown	16·60	16·67	+0·07	16·70	16·63	+0·07
Dunmore East ..	15·80	15·88	+0·08	15·90	15·79	+0·11
New Ross	16·74	16·68	-0·06	16·85	16·61	+0·24
Passage West....	15·70	15·77	+0·07	15·77	15·70	+0·07
Castle Townsend..	15·42	15·47	+0·05	15·46	15·44	+0·02

Upon comparing the results of this Table with those of the Table on page 96, the remarks made in page 33 must, I think, be considered to be insufficiently founded. The excess found by classifying according to the magnitude of the tide is here decidedly greater than that found by classifying according to the moon's declination. In the river tides (from Kilbaha to Limerick, and from Dunmore East to New Ross) the excess classified by the magnitude of tide proceeds more regularly than that classified by moon's declination. I think also that the change for the stations on the narrow channel from Port Rush to Donaghadee inclines us to the supposition that the whole is due rather to the variation of magnitude than to the variation of declination. The change of declination, being very slow, would probably produce the same sensible change in the mean level of the deep (though contracted) *sea* of the North Channel as in that of the Atlantic Ocean: though everything which depended in any way upon the tides might be very different. Perhaps however the change for these stations is not sufficiently decided to give great force to this argument. On the whole, I regard the origin of this inequality as yet subject to some doubt.

The values of A_0 at any one station differ, sometimes rapidly, from day to day. In order to examine these, I have subtracted from every value of A_0 at each place the

mean of all the values of A_0 at that place, and have set down the excess as an irregularity in the general height of the water at that place on that day. The numbers for Courtown are taken by anticipation from Section XVII. The results are contained in the following Table. With the view of ascertaining any possible connexion of these irregularities with the cause of the winds, I have set down in the last columns the character of the winds observed at the four stations Kilbaha, Port Rush, Kingstown, and Passage West, which may be considered as nearly equidistant on the coast. [The letter *c* attached to the letters describing the direction of the wind denotes that it was nearly calm ; the letter *s* denotes that the wind was strong.]

Wind.

Excess of each value of A₀ above the mean value of A₀ at every station.

Approximate time of high water at Kilbaha.	Excess of each value of A ₀ above the mean value of A ₀ at every station.																Wind.													
	Kilbaha.	Kilrush.	Foynes Island.	Lime-riek.	Casleh Bay.	Galway.	Old Head.	Mul-high-moore.	Bun-cran.	Port Rush.	Carrow-keel.	Bally-castle.	Glen-arm.	Donaghadee.	Ard-glass.	Clogher Head.	Kings-town.	Cour-town.	Dum-more East.	New Ross.	Passage West.	Castle Towns- end.	Kilbaha.	Port Rush.	Kingstown.	Passage West.				
July 23. 18	ft. -0.18	ft. -0.13	ft. -0.19	ft. -0.10	ft. -0.05	ft. -0.23	ft. -0.23	ft. -0.22	ft. -0.19	ft. -0.15	ft. -0.15	ft. -0.23	ft. -0.46	ft. -0.29	ft. -0.24	ft. -0.29	ft. -0.20	ft. -0.31	ft. -0.11	ft. -0.15	ft. -0.02	ft. 0.00	N.E.C.	N.E.	N.E.C.	N.W.	N.W.			
24. 19	ft. -0.07	ft. -0.08	ft. -0.12	ft. -0.23	ft. -0.12	ft. -0.24	ft. -0.08	ft. -0.14	ft. -0.14	ft. -0.20	ft. -0.11	ft. -0.27	ft. -0.40	ft. -0.28	ft. -0.24	ft. -0.02	ft. -0.30	ft. -0.12	ft. -0.01	ft. -0.15	ft. 0.00	ft. -0.02	N.W.C.	N.W.	N.W.C.	N.W.	N.W.			
25. 20	ft. -0.17	ft. -0.23	ft. -0.25	ft. -0.31	ft. -0.27	ft. -0.34	ft. -0.29	ft. -0.51	ft. -0.44	ft. -0.35	ft. -0.28	ft. -0.41	ft. -0.39	ft. -0.26	ft. -0.20	ft. -0.27	ft. -0.20	ft. -0.26	ft. -0.06	ft. -0.15	ft. -0.09	ft. -0.06	N.W.C.	N.W.	N.W.C.	N.W.	N.W.			
26. 21	ft. -0.34	ft. -0.46	ft. -0.41	ft. -0.55	ft. -0.42	ft. -0.60	ft. -0.38	ft. -0.50	ft. -0.53	ft. -0.50	ft. -0.52	ft. -0.54	ft. -0.58	ft. -0.40	ft. -0.32	ft. -0.32	ft. -0.24	ft. -0.22	ft. -0.22	ft. -0.18	ft. -0.26	ft. -0.15	ft. -0.07	E.	N.W.	N.W.C.	N.W.	N.W.		
27. 22	ft. -0.37	ft. -0.49	ft. -0.40	ft. -0.46	ft. -0.41	ft. -0.58	ft. -0.38	ft. -0.36	ft. -0.36	ft. -0.34	ft. -0.47	ft. -0.29	ft. -0.51	ft. -0.25	ft. -0.25	ft. -0.15	ft. -0.23	ft. -0.21	ft. -0.21	ft. -0.26	ft. -0.38	ft. -0.34	ft. -0.25	N.W.C.	W.N.W.S.	N.W.S.	N.W.	N.W.		
28. 23	ft. -0.48	ft. -0.64	ft. -0.58	ft. -0.79	ft. -0.54	ft. -0.54	ft. -0.54	ft. -0.56	ft. -0.58	ft. -0.66	ft. -0.51	ft. -0.46	ft. -0.77	ft. -0.55	ft. -0.43	ft. -0.24	ft. -0.31	ft. -0.19	ft. -0.19	ft. -0.29	ft. -0.54	ft. -0.21	ft. -0.09	N.W.C.	W.N.W.S.	N.W.S.	N.W.	N.W.		
29. 24	ft. -0.33	ft. -0.42	ft. -0.40	ft. -0.44	ft. -0.36	ft. -0.47	ft. -0.45	ft. -0.49	ft. -0.38	ft. -0.54	ft. -0.57	ft. -0.35	ft. -0.67	ft. -0.44	ft. -0.37	ft. -0.32	ft. -0.30	ft. -0.27	ft. -0.27	ft. -0.17	ft. -0.50	ft. -0.18	ft. -0.14	N.W.C.	N.W.	N.W.C.	N.W.	N.W.		
30. 25	ft. -0.40	ft. -0.46	ft. -0.50	ft. -0.71	ft. -0.48	ft. -0.52	ft. -0.51	ft. -0.63	ft. -0.34	ft. -0.32	ft. -0.38	ft. -0.41	ft. -0.40	ft. -0.44	ft. -0.38	ft. -0.45	ft. -0.35	ft. -0.32	ft. -0.32	ft. -0.32	ft. -0.58	ft. -0.30	ft. -0.16	N.W.C.	N.W.	N.W.C.	N.W.	N.W.		
31. 26	ft. -0.20	ft. -0.30	ft. -0.29	ft. -0.54	ft. -0.27	ft. -0.31	ft. -0.24	ft. -0.42	ft. -0.16	ft. -0.29	ft. -0.10	ft. -0.45	ft. -0.26	ft. -0.28	ft. -0.34	ft. -0.27	ft. -0.28	ft. -0.42	ft. -0.42	ft. -0.19	ft. -0.39	ft. -0.10	ft. -0.04	N.W.C.	N.W.	N.W.C.	N.W.	N.W.		
August 1. 12	ft. +0.10	ft. +0.15	ft. +0.02	ft. -0.21	ft. +0.12	ft. +0.10	ft. +0.05	ft. +0.11	ft. +0.28	ft. +0.12	ft. -0.09	ft. -0.03	ft. +0.30	ft. +0.14	ft. -0.01	ft. +0.03	ft. +0.01	ft. -0.06	ft. -0.06	ft. +0.06	ft. -0.08	ft. +0.11	ft. +0.16	N.W.C.	W.S.W.S.	N.E.C.	N.W.	N.W.		
2. 14	ft. -0.27	ft. -0.12	ft. -0.18	ft. -0.53	ft. -0.25	ft. -0.19	ft. -0.25	ft. -0.20	ft. -0.16	ft. -0.38	ft. -0.28	ft. -0.21	ft. +0.12	ft. -0.05	ft. -0.14	ft. +0.12	ft. -0.14	ft. -0.21	ft. -0.21	ft. -0.11	ft. -0.34	ft. -0.10	ft. -0.02	N.W.	N.W.	N.W.C.	N.W.	N.W.		
3. 15	ft. -0.20	ft. -0.10	ft. -0.02	ft. 0.00	ft. -0.06	ft. +0.06	ft. -0.16	ft. +0.20	ft. +0.05	ft. +0.15	ft. +0.15	ft. +0.26	ft. +0.35	ft. +0.15	ft. +0.13	ft. +0.23	ft. +0.05	ft. -0.15	ft. -0.16	ft. -0.14	ft. -0.24	ft. -0.23	ft. -0.11	N.W.	N.W.	N.W.C.	N.W.	N.W.		
4. 16	ft. +0.03	ft. +0.20	ft. +0.20	ft. +0.44	ft. +0.19	ft. +0.19	ft. +0.52	ft. +0.27	ft. +0.39	ft. +0.44	ft. +0.32	ft. +0.54	ft. +0.60	ft. +0.41	ft. +0.31	ft. +0.31	ft. +0.28	ft. +0.10	ft. +0.10	ft. +0.04	ft. +0.08	ft. -0.08	ft. -0.05	N.W.	W.S.W.S.	N.W.S.	N.W.	N.W.		
5. 17	ft. +0.05	ft. +0.24	ft. +0.20	ft. +0.37	ft. +0.17	ft. +0.49	ft. +0.40	ft. +0.51	ft. +0.37	ft. +0.28	ft. +0.36	ft. +0.35	ft. +0.36	ft. +0.16	ft. +0.19	ft. +0.18	ft. +0.12	ft. +0.05	ft. +0.05	ft. +0.08	ft. +0.04	ft. +0.07	ft. +0.06	W.S.W.S.	W.N.W.S.	N.W.S.	N.W.	N.W.		
6. 18	ft. +0.15	ft. +0.36	ft. +0.35	ft. +0.76	ft. +0.31	ft. +0.46	ft. +0.60	ft. +0.42	ft. +0.42	ft. +0.47	ft. +0.52	ft. +0.59	ft. +0.58	ft. +0.35	ft. +0.33	ft. +0.27	ft. +0.23	ft. +0.02	ft. +0.10	ft. +0.31	ft. +0.04	ft. +0.06	ft. +0.06	N.W.	W.S.W.S.	N.E.C.	N.W.	N.W.		
7. 19	ft. +0.24	ft. +0.32	ft. +0.59	ft. +0.34	ft. +0.37	ft. +0.37	ft. +0.37	ft. 0.00	ft. +0.27	ft. +0.34	ft. +0.53	ft. +0.31	ft. +0.43	ft. +0.05	ft. +0.11	ft. -0.10	ft. -0.09	ft. -0.17	ft. +0.03	ft. +0.41	ft. +0.03	ft. +0.02	ft. +0.02	S.	S.E.S.	E.	S.W.	S.W.		
8. 19	ft. +0.29	ft. +0.25	ft. +0.38	ft. +0.42	ft. +0.20	ft. +0.21	ft. +0.18	ft. +0.17	ft. +0.31	ft. +0.31	ft. +0.35	ft. +0.36	ft. +0.45	ft. +0.19	ft. +0.16	ft. -0.08	ft. +0.15	ft. +0.01	ft. +0.16	ft. +0.39	ft. +0.20	ft. +0.20	ft. +0.20	S.C.	S.W.	E.	S.W.	S.W.		
9. 20	ft. +0.11	ft. +0.07	ft. +0.30	ft. +0.51	ft. +0.10	ft. +0.06	ft. +0.12	ft. +0.41	ft. +0.31	ft. +0.19	ft. +0.16	ft. +0.06	ft. +0.52	ft. +0.20	ft. +0.13	ft. +0.11	ft. +0.14	ft. +0.68	ft. +0.17	ft. +0.40	ft. +0.14	ft. +0.14	ft. +0.14	N.C.	N.W.C.	S.W.C.	S.S.	S.W.		
10. 21	ft. -0.02	ft. -0.08	ft. -0.05	ft. +0.08	ft. -0.02	ft. 0.00	ft. -0.14	ft. -0.16	ft. -0.03	ft. -0.15	ft. -0.28	ft. -0.18	ft. +0.33	ft. +0.10	ft. +0.09	ft. +0.28	ft. +0.03	ft. -0.07	ft. -0.15	ft. +0.03	ft. -0.11	ft. +0.04	ft. +0.04	W.S.W.S.	W.S.W.	N.W.S.	N.W.S.	N.W.S.		
11. 9	ft. -0.01	ft. -0.06	ft. +0.04	ft. +0.12	ft. +0.01	ft. -0.10	ft. -0.10	ft. -0.11	ft. +0.11	ft. +0.11	ft. -0.31	ft. -0.01	ft. +0.32	ft. +0.10	ft. +0.16	ft. 0.00	ft. +0.09	ft. -0.01	ft. -0.11	ft. +0.14	ft. -0.07	ft. -0.12	ft. -0.12	N.W.C.	N.W.	N.W.C.	N.W.	N.W.		
12. 10	ft. -0.24	ft. -0.27	ft. -0.25	ft. -0.47	ft. -0.15	ft. -0.35	ft. -0.23	ft. -0.11	ft. -0.08	ft. -0.14	ft. -0.45	ft. +0.07	ft. +0.15	ft. -0.07	ft. -0.02	ft. +0.01	ft. -0.11	ft. 0.00	ft. 0.00	ft. +0.39	ft. +0.20	ft. +0.20	ft. +0.20	W.S.W.S.	W.S.W.S.	S.E.S.	S.S.	S.W.		
13. 11	ft. -0.38	ft. -0.46	ft. -0.46	ft. -0.78	ft. -0.40	ft. -0.36	ft. -0.36	ft. -0.34	ft. -0.31	ft. -0.33	ft. -0.55	ft. +0.02	ft. -0.20	ft. -0.33	ft. -0.30	ft. +0.10	ft. -0.30	ft. -0.34	ft. -0.34	ft. -0.44	ft. -0.64	ft. -0.44	ft. -0.26	ft. -0.26	S.W.C.	S.W.C.	S.W.C.	S.W.	S.W.	
14. 12	ft. -0.38	ft. -0.37	ft. -0.46	ft. -0.79	ft. -0.34	ft. -0.44	ft. -0.47	ft. -0.49	ft. -0.44	ft. -0.49	ft. -0.47	ft. -0.24	ft. -0.29	ft. -0.37	ft. -0.36	ft. -0.10	ft. -0.25	ft. -0.34	ft. -0.34	ft. -0.27	ft. -0.48	ft. -0.29	ft. -0.22	ft. -0.22	S.W.C.	S.W.C.	S.W.C.	S.W.	S.W.	
15. 13	ft. -0.24	ft. -0.20	ft. -0.41	ft. -0.80	ft. -0.25	ft. -0.33	ft. -0.28	ft. -0.43	ft. -0.41	ft. -0.51	ft. -0.30	ft. -0.36	ft. -0.39	ft. -0.48	ft. -0.42	ft. -0.15	ft. -0.34	ft. -0.28	ft. -0.28	ft. -0.26	ft. -0.46	ft. -0.17	ft. -0.12	ft. -0.12	N.W.C.	N.W.	N.W.C.	N.W.	N.W.	
16. 14	ft. -0.06	ft. +0.09	ft. -0.12	ft. -0.45	ft. -0.03	ft. -0.01	ft. -0.01	ft. -0.14	ft. +0.06	ft. -0.04	ft. +0.02	ft. 0.00	ft. 0.00	ft. -0.02	ft. -0.01	ft. +0.22	ft. +0.16	ft. +0.15	ft. +0.17	ft. -0.22	ft. +0.09	ft. +0.13	ft. +0.13	ft. +0.13	E.	N.W.C.	N.E.C.	N.W.	N.W.	
17. 3	ft. -0.05	ft. -0.05	ft. -0.07	ft. -0.32	ft. +0.01	ft. -0.13	ft. -0.11	ft. -0.14	ft. -0.03	ft. -0.10	ft. 0.00	ft. -0.02	ft. -0.03	ft. -0.03	ft. -0.06	ft. -0.03	ft. +0.02	ft. -0.05	ft. +0.20	ft. -0.11	ft. +0.24	ft. +0.29	ft. +0.29	ft. +0.29	S.E.	N.W.C.	N.E.C.	N.W.	N.W.	
18. 4	ft. +0.11	ft. +0.11	ft. +0.10	ft. +0.08	ft. +0.14	ft. +0.23	ft. +0.10	ft. +0.13	ft. +0.14	ft. +0.16	ft. +0.17	ft. +0.25	ft. +0.21	ft. +0.15	ft. +0.19	ft. +0.24	ft. +0.20	ft. +0.44	ft. +0.32	ft. +0.13	ft. +0.26	ft. +0.26	ft. +0.16	ft. +0.16	N.W.C.	N.W.C.	S.	N.W.	N.W.	
19. 5	ft. +0.09	ft. +0.09	ft. +0.07	ft. +0.19	ft. +0.14	ft. +0.10	ft. +0.27	ft. +0.39	ft. +0.30	ft. +0.44	ft. +0.26	ft. +0.46	ft. +0.46	ft. +0.29	ft. +0.27	ft. +0.44	ft. +0.17	ft. +0.46	ft. +0.11	ft. +0.09	ft. +0.05	ft. +0.13	ft. +0.13	ft. +0.13	S.	N.W.C.	S.E.C.	N.W.	N.W.	
20. 6	ft. -0.11	ft. -0.13	ft. -0.14	ft. -0.07	ft. -0.16	ft. -0.19	ft. -0.62	ft. -0.52	ft. -0.07	ft. -0.12	ft. -0.14	ft. -0.14	ft. -0.05	ft. -0.18	ft. -0.15	ft. -0.41	ft. -0.10	ft. -0.06	ft. -0.06	ft. -0.15	ft. -0.10	ft. -0.02	ft. -0.02	ft. -0.02	N.W.C.	N.W.C.	N.E.C.	N.W.	N.W.	
21. 18	ft. -0.41	ft. -0.44	ft. -0.45	ft. -0.47	ft. -0.44	ft. -0.50	ft. -0.13	ft. -0.24	ft. -0.29	ft. -0.36	ft. -0.41	ft. -0.47	ft. -0.49	ft. -0.58	ft. -0.57	ft. -0.50	ft. -0.63	ft. -0.37	ft. -0.37	ft. -0.56	ft. -0.44	ft. -0.26	ft. -0.26	ft. -0.26	N.W.C.	N.W.C.	N.E.C.	N.W.	N.W.	
22. 6	ft. -0.07	ft. -0.02	ft. -0.08	ft. -0.02	ft. -0.07	ft. -0.07	ft. -0.11	ft. -0.27	ft. +0.27	ft. +0.17	ft. +0.03	ft. +0.05	ft. +0.02	ft. -0.10	ft. -0.10	ft. -0.10	ft. -0.05	ft. -0.05	ft. -0.17	ft. +0.10	ft. +0.10	ft. +0.08	ft. +0.03	ft. +0.03	N.W.C.	N.W.C.	N.E.C.	N.W.	N.W.	
23. 7																										S.W.	N.W.C.	E.C.	N.W.	N.W.

Upon inspecting these numbers, one law cannot fail to occur to us, namely, that the irregularities are nearly the same in magnitude and in sign on every part of the coast of Ireland at the same time. So prevalent is this law, that there are few instances in which the irregularity at one place differs from the mean of the irregularities at all the places at the same time by more than an inch. My ideas of the almost perfect fluidity of water have been very much raised by this comparison. I may remark that it embodies, in a form admitting of easy examination, the result concluded from rough inspection of observations which is made the foundation of a method for supplying deficient observations described in page 10. I may also add that it gives no small security for the general fidelity and accuracy of the observers at the different stations.

I do not perceive any certain connexion between the irregularities and the course of the winds, except that the water is usually highest on all parts of the coast with a violent south-west wind.

In order to ascertain the general relation of these irregularities to those of the barometer, I have compared the corrected mean height of the barometer at Greenwich for each civil day with the mean height for the year (using the numbers published in the Greenwich Magnetical and Meteorological Observations). The following are the results:—

Excess of the corrected mean height of the barometer each day, at Greenwich, above the mean height for the year 1842.											
	in.		in.		in.		in.		in.		in.
June 23.	-0.142	July 4.	-0.226	July 15.	+0.378	July 26.	+0.042	Aug. 6.	-0.033	Aug. 17.	+0.132
24.	-0.251	5.	-0.232	16.	+0.174	27.	+0.187	7.	18.	-0.041
25.	-0.234	6.	+0.129	17.	28.	+0.056	8.	+0.078	19.	-0.067
26.	7.	+0.070	18.	-0.118	29.	-0.069	9.	+0.017	20.	+0.032
27.	+0.218	8.	-0.216	19.	-0.147	30.	+0.083	10.	-0.263	21.
28.	+0.268	9.	-0.271	20.	-0.231	31.	11.	-0.029	22.	-0.023
29.	+0.097	10.	21.	-0.189	Aug. 1.	+0.310	12.	+0.313	23.	-0.064
30.	-0.080	11.	-0.288	22.	+0.064	2.	+0.164	13.	+0.443		
July 1.	-0.168	12.	-0.046	23.	+0.235	3.	-0.016	14.		
2.	-0.105	13.	+0.226	24.	4.	-0.092	15.	+0.242		
3.	14.	+0.388	25.	-0.040	5.	-0.036	16.	+0.207		

I have also obtained, through the kindness of Sir W. R. HAMILTON, the barometrical observations at the Observatory of Dunsink, near Dublin; and from Colonel COLBY and Captain LARCOM, R.E., I have received the observations made in the Phoenix Park near Dublin, and at Limerick. The following results are obtained by comparing each day's mean with the mean for this period.

Excess of the mean height of the barometer each day, at Dublin, above the mean of all.																	
	Dun-sink.	Phoenix Park.		Dun-sink.	Phoenix Park.		Dun-sink.	Phoenix Park.		Dun-sink.	Phoenix Park.		Dun-sink.	Phoenix Park.		Dun-sink.	Phoenix Park.
June 23.	in. -0.31	in. -0.372	July 4.	in. -0.53	in. -0.547	July 15.	in. +0.35	in. +0.388	July 26.	in. +0.19	in. +0.164	Aug. 6.	in. -0.04	in. -0.070	Aug. 17.	in. +0.09	in. +0.059
24.	-0.42	-0.454	5.	-0.22	-0.281	16.	+0.04	+0.076	27.	+0.31	+0.302	7.	-0.112	18.	+0.01	-0.013
25.	-0.49	-0.534	6.	+0.06	+0.173	17.	-0.196	28.	+0.20	+0.182	8.	-0.08	-0.097	19.	-0.17	-0.210
26.	+0.06	-0.098	7.	-0.22	-0.227	18.	-0.13	-0.153	29.	+0.20	+0.173	9.	-0.10	-0.111	20.	-0.08	-0.105
27.	+0.31	+0.315	8.	-0.38	-0.500	19.	-0.12	-0.165	30.	+0.29	+0.284	10.	-0.21	-0.258	21.	+0.12	+0.115
28.	+0.16	+0.148	9.	-0.32	-0.365	20.	-0.11	-0.147	31.	+0.365	11.	-0.01	-0.011	22.	+0.17	+0.165
29.	+0.13	+0.120	10.	-0.25	-0.180	21.	+0.01	-0.006	Aug. 1.	+0.32	+0.315	12.	+0.17	+0.170	23.	-0.07	-0.079
30.	+0.02	+0.022	11.	-0.43	-0.485	22.	+0.22	+0.205	2.	+0.09	+0.072	13.	+0.42	+0.418			
July 1.	-0.10	-0.126	12.	-0.09	-0.074	23.	+0.30	+0.304	3.	+0.10	+0.089	14.	+0.445			
2.	-0.04	-0.059	13.	+0.14	+0.121	24.	+0.200	4.	-0.09	-0.110	15.	+0.37	+0.342			
3.	-0.25	-0.108	14.	+0.45	+0.443	25.	+0.09	+0.061	5.	-0.14	-0.176	16.	+0.29	+0.284			

Excess of the mean height of the barometer each day, at Limerick, above the mean of all.											
	in.		in.		in.		in.		in.		in.
June 23.	-0.407	July 4.	-0.543	July 15.	+0.365	July 26.	+0.182	Aug. 6.	-0.062	Aug. 17.	+0.058
24.	-0.447	5.	-0.169	16.	-0.059	27.	+0.324	7.	-0.169	18.	-0.002
25.	-0.535	6.	+0.212	17.	-0.320	28.	+0.295	8.	-0.171	19.	-0.198
26.	-0.077	7.	-0.217	18.	-0.208	29.	+0.220	9.	-0.152	20.	-0.115
27.	+0.360	8.	-0.514	19.	-0.178	30.	+0.337	10.	-0.267	21.	+0.099
28.	+0.178	9.	-0.346	20.	-0.132	31.	+0.381	11.	-0.053	22.	+0.190
29.	+0.178	10.	-0.191	21.	+0.030	Aug. 1.	+0.323	12.	+0.122	23.	-0.069
30.	+0.039	11.	-0.491	22.	+0.173	2.	+0.052	13.	+0.406		
July 1.	-0.072	12.	-0.104	23.	+0.308	3.	+0.124	14.	+0.422		
2.	+0.022	13.	+0.094	24.	+0.206	4.	-0.051	15.	+0.360		
3.	-0.116	14.	+0.440	25.	+0.074	5.	-0.162	16.	+0.301		

The comparison of these numbers with the irregularities in the heights of the water amply supports the law of DAUSSY, WHEWELL, and BUNT, that a negative irregularity in the height of the barometer is accompanied by a positive irregularity in the height of the sea, twelve or fourteen times as great as that of the barometer.

Section XII.—*Discussion of range of tide, or coefficient of first arc in the analysis of individual tides; and of semimenstrual inequality in range, apparent proportion of solar and lunar effects, and age of tide as deduced from range.*

The tides were divided into groups of large tides and small tides, separated at the same times as those particularized in pages 29 and 96. For each of these groups the mean of the values of C_1 was taken, some deficient values being supplied by interpolation. The moon's parallax and the square of the cosine of the moon's and sun's declinations were taken for two days preceding each tide, and the means of these quantities were taken through the same groups.

A general result will be obtained by forming the sum of the mean values of C_1 for all the stations in each group. Thus we obtain:—

Large Tides.				
Period.	Sum of mean values of C_1 at all the stations.	Mean value of \cos^2 moon's declination.	Mean value of moon's parallax.	Mean value of \cos^2 sun's declination.
July 6. 16 to July 13. 3	ft. 117.43	0.8742	59 44	0.8547
July 20. 9 to July 28. 9	100.11	0.9076	54 41	0.8815
Aug. 5. 0 to Aug. 11. 16	129.11	0.9260	60 31	0.9194
Small Tides.				
Period.	Sum of mean values of C_1 at all the stations.	Mean value of \cos^2 moon's declination.	Mean value of moon's parallax.	Mean value of \cos^2 sun's declination.
June 28. 6 to July 6. 16	ft. 71.66	0.9427	55 13	0.8456
July 13. 3 to July 20. 9	84.63	0.9202	58 19	0.8666
July 28. 9 to Aug. 5. 0	73.40	0.8965	55 56	0.9012
Aug. 11. 16 to Aug. 18. 15	77.84	0.8697	57 34	0.9378

Put M for the lunar effect when the square of the cosine of the moon's declination is $0\cdot9$ and her parallax is $57'$; m for the quantity by which this is increased for every increase of $1'$ in parallax; s for the mean effect of the sun (the square of the cosine of his declination being $0\cdot9$) for one-fourth of a lunation, positive in the large tides and negative in the small tides, which bears to the absolute effect of the sun a relation explained in page 34. The effect of the variations of the moon's parallax and declination upon the luni-solar tide, as is well known, is nearly the same as that on the simple lunar tide; and therefore it will be correct to refer the mean of the luni-solar tides to the mean of the moon's parallaxes and square of cosine of declinations. The variations depending on the moon's declinations are not strictly in the proportion of the squares of the cosines of her declination, but in the present instance, where the means of the squares of the cosines are very nearly equal, may be assumed to be so without sensible error. Forming then an equation from each of the lines in the Table above by these considerations; reducing them to four equations by retaining the second, taking the mean of the first and third, the mean of the fourth and sixth, and the mean of the fifth and seventh, and combining these so as to form three favourable equations, by adding all, by subtracting the sum of the third and fourth from the sum of the others, and by subtracting the sum of the second and third from the sum of the others, we obtain the following equations:—

$$\begin{aligned} 377\cdot15 &= M \times 4\cdot024 + m \times 0\cdot33 - s \times 0\cdot008 \\ 69\cdot61 &= -M \times 0\cdot008 + m \times 1\cdot29 + s \times 3\cdot938 \\ 31\cdot87 &= -M \times 0\cdot036 + m \times 7\cdot83 - s \times 0\cdot026. \end{aligned}$$

From these we obtain $M=93\cdot4$, $m=4\cdot56$, $s=16\cdot37$. The moon's effect, therefore, for the parallax $57' + n'$ may be represented by $93\cdot4 + 4\cdot56 \times n$. If the moon's hydrodynamical effect varied as the cube of her parallax (which is the law of variation of her statical effect), the formula would be $93\cdot4 + 4\cdot92 \times n$. The result of the movement of the water has therefore been, to reduce the elliptic variation of lunar effect by $\frac{0\cdot36}{4\cdot92}$ part, or by $\frac{3}{41}$ part.

Now it is shown in the *Encyclopædia Metropolitana*, *Tides and Waves*, Art. 448, that if the tides were created by the effect of the moon on the water in a uniform channel surrounding the earth, and if b were the earth's radius, k the depth of the water, g the acceleration produced by gravity in the unit of time, n' the moon's apparent angular motion round the earth (as estimated by a spectator who supposes that the earth does not revolve on an axis), and h the moon's angular motion from her perigee; then the elliptic variation is changed by $\frac{4}{3} \cdot \frac{n'b^2h}{n'^2b^2 - gk}$ part. Thus we obtain

$$\frac{4}{3} \cdot \frac{n'b^2h}{n'^2b^2 - gk} = -\frac{3}{41}.$$

Making $\frac{h}{n} = \frac{3}{80}$, we find $\frac{gk}{n'^2b^2} = \frac{101}{60} = \frac{5}{3}$ nearly; and $k = \frac{5}{3} \cdot \frac{n'^2b^2}{g}$. Observing that $n'b$

=linear velocity of a point at the equator produced by the earth's rotation, supposing the moon fixed, we easily find $\frac{n^2 b^2}{g} = 13$ miles nearly; and thus our observations give

$$\text{Depth of the sea} = k = 22 \text{ miles.}$$

If the channel were supposed to be a small circle of the earth instead of a large one, the resulting depth of the sea would be diminished in the proportion of the square of its diameter.

Whatever may be supposed of the error of this result, or the inapplicability of the theory by which it is obtained to the circumstances of the seas, I may remark that it agrees generally with a result deduced from Mr. WHEWELL's discussions of the observations at Bristol with reference to the moon's declinations*.

Assuming however that we have correctly determined $\frac{n^2 b^2}{gk} = \frac{3}{5}$, we may proceed to remark that † the moon's hydrodynamical effect is represented by her statical effect multiplied by $\frac{1}{gk - n^2 b^2}$ and by constants; and that the sun's hydrodynamical effect is represented by his statical effect multiplied by $\frac{1}{gk - n^2 b^2}$ and by the same constants. If we consider $\frac{n^2}{n^2} = \frac{15}{14}$, then the hydrodynamical effect of the moon contains the multiplier

$$\frac{1}{gk} \cdot \frac{1}{1 - \frac{3}{5}} = \frac{1}{gk} \cdot \frac{5}{2} \text{ or } \frac{1}{gk} \cdot \frac{25}{10},$$

while that for the sun contains the multiplier

$$\frac{1}{gk} \cdot \frac{1}{1 - \frac{15}{14} \cdot \frac{3}{5}} = \frac{1}{gk} \cdot \frac{14}{5} \text{ or } \frac{1}{gk} \cdot \frac{28}{10}.$$

And therefore the proportion of the moon's statical effect to the sun's is greater than the proportion of her dynamical effect to the sun's in the ratio of 28 to 25. And as the moon's hydrodynamical effect, deduced from the values of M and s above (93.4 and 16.37), by the considerations in page 34, is nearly $= \frac{1.00}{0.28} \times$ sun's hydrodynamical effect, it follows that the moon's statical effect $= \frac{1.00}{0.25} \times$ sun's statical effect $= 4 \times$ sun's statical effect. This conclusion differs widely from LAPLACE's; yet it is formed, as I believe, on grounds as good as LAPLACE's.

For particular results applying to each individual station, regarding the semi-menstrual inequality in range and the apparent proportion of the solar and lunar hydrodynamical effects; the mean value of C_1 for large tides is found by taking the mean of the three values in the three periods of the last Table, and the mean value for small tides by taking the mean of the four values in the four periods of the last

* Tides and Waves, Art. 553.

† Ibid. Art. 448.

Table; and then treating these in the same manner as in the latter part of the table on page 35.

Station.	Mean of C_1 for large tides.	Mean of C_1 for small tides.	Difference.	Mean, or M.	Difference divided by mean.	Corresponding value of $\frac{S}{M}$.
	ft.	ft.	ft.	ft.		
Kilbaha	6.12	3.73	2.39	4.93	0.48	0.40
Kilrush	6.54	4.19	2.35	5.37	0.44	0.36
Foynes Island.	7.35	4.96	2.39	6.16	0.39	0.32
Limerick.	8.98	6.21	2.77	7.59	0.37	0.30
Casleh Bay.	6.35	3.83	2.52	5.09	0.50	0.42
Galway	6.41	3.89	2.52	5.15	0.49	0.41
Old Head	5.59	3.43	2.16	4.51	0.48	0.40
Mullaghmore	5.35	3.25	2.10	4.30	0.49	0.41
Buncrana	5.84	3.37	2.47	4.61	0.54	0.46
Port Rush	2.56	1.38	1.18	1.97	0.60	0.52
Carrowkeel.	3.35	1.95	1.40	2.65	0.53	0.45
Ballycastle	1.42	0.88	0.54	1.15	0.47	0.39
Glenarm	2.89	2.47	0.42	2.68	0.16	0.12
Donaghadee	5.24	3.96	1.28	4.60	0.28	0.22
Ardglass.	6.90	4.96	1.94	5.93	0.33	0.26
Clogher Head	6.79	4.81	1.98	5.80	0.34	0.27
Kingstown	5.26	3.69	1.57	4.48	0.35	0.28
Dunmore East	5.73	3.91	1.82	4.82	0.38	0.31
New Ross	6.16	4.51	1.65	5.34	0.31	0.25
Passage West.	5.94	4.19	1.75	5.07	0.35	0.28
Castle Townsend.	4.79	3.33	1.46	4.06	0.36	0.29

For the age of tide as inferred from range, the times have been ascertained (by interpolating between the times in the Tables of formulæ in Section X.) at which the actual value of C_1 may be supposed to coincide with the mean value of C_1 ; and the times thus found have been compared with the times at which the moon's hour-angle from the sun was 3^h , 9^h , 15^h , 21^h , namely, June 26, 6^h , July 4, 16^h , July 11, 3^h , July 18, 9^h , July 26, 9^h , August 3, 0^h , August 9, 16^h , and August 16, 15^h . An error is here committed alternately + and -, and therefore it is proper to use an even number of comparisons. Eight are used at every place except Ballycastle and Glenarm, where only six are used. [It is to be remarked that in the use of the formulæ in Section X., a number opposite to a bracket is always held to correspond to the mean of the two times embraced by that bracket.] The means of all the differences at each station between the times thus found from Section X. and the times corresponding to the hour-angles 3^h , 9^h , &c., are adopted in the following Table as the age of the tide. These* are the true ages of the tide.

Station.	Age of tide.		Station.	Age of tide.		Station.	Age of tide.	
	d	h		d	h		d	h
Kilbaha	1	20	Mullaghmore.	2	0	Ardglass.	2	2
Kilrush	1	19	Buncrana	1	20	Clogher Head	2	2
Foynes Island.	1	21	Port Rush	1	11	Kingstown	2	0
Limerick.	2	1	Carrowkeel.	1	14	Dunmore East	2	2
Casleh Bay.	1	20	Ballycastle	1	5	New Ross	2	2
Galway	1	21	Glenarm.	2	9	Passage West.	2	1
Old Head	1	22	Donaghadee	2	6	Castle Townsend.	1	23

* Tides and Waves, Art. 545.

Section XIII.—*Establishment of each port, as deduced from the time of maximum of the first periodical term in the analysis of individual tides.*

The preceding operations having made it sufficiently clear that the age of the tide differs little from two days, it is proper now to refer all the phenomena of the tide to an epoch two days earlier than the observation. A process is therefore adopted in this section differing in a trifling degree from that in Section VII. The times are taken at which the moon's hour-angle from the sun was 0^h, 6^h, 12^h, 18^h, and two days are added to these times: the resulting times are June 24, 9^h, July 2, 20^h, July 9, 19^h, July 16, 21^h, July 23, 23^h, August 1, 16^h, August 8, 4^h, August 15, 5^h, August 22, 16^h. These times are assumed to divide the large lunitidal intervals from the small ones. For the lunitidal intervals at the high waters of the First Division, the moon's time of Greenwich transit is taken for that transit which precedes the high water by two days and a few hours; for the high waters of the Second Division the moon's time of Greenwich lower transit is interpolated. Comparing these times of transit with the times of high water in the formulæ of Section X., the lunitidal intervals corresponding to lunar transits two days earlier are found. The means of these are taken for the groups separated by the times specified above. Then, the means of the large and the small lunitidal intervals being adopted as the mean interval, a correction of 1^h 41^m is applied subtractively, to reduce the interval after moon's transit two days previous, to the interval after moon's transit on the same day as the tide (1^h 41^m being the difference between two solar days and two lunar days). The result is contained in the following Table.

Station.	Mean of large intervals from transit two days previous.		Mean of small intervals from transit two days previous.		Mean of all the intervals from transit two days previous.		True establishment in Greenwich time.	
	h	m	h	m	h	m	h	m
Kilbaha	6	53	5	55	6	24	4	43
Kilrush	7	6	6	7	6	36	4	55
Foynes Island.	7	49	6	54	7	21	5	40
Limerick.	8	19	7	27	7	53	6	12
Casleh Bay.	7	3	6	3	6	33	4	52
Galway	7	8	6	9	6	38	4	57
Old Head	7	19	6	23	6	51	5	10
Mullaghmore.	7	39	6	37	7	8	5	27
Buncrana	8	27	7	21	7	54	6	13
Port Rush	9	4	7	50	8	27	6	46
Carrowkeel.	9	48	8	44	9	16	7	35
Ballycastle	10	4	9	2	9	33	7	52
Glenarm	12	53	12	3	12	28	10	47
Donaghadee.	13	10	12	16	12	43	11	2
Ardglass.	13	14	12	16	12	45	11	4
Clogher Head.	13	21	12	24	12	52	11	11
Kingstown	13	26	12	30	12	58	11	17
Dunmore East.	7	14	6	16	6	45	5	4
New Ross	8	5	7	11	7	38	5	57
Passage West.	7	26	6	31	6	58	5	17
Castle Townsend.	7	3	6	5	6	34	4	53

Section XIV.—*Semimenstrual inequality in time, proportion of solar and lunar effects from times, and apparent age of tide as shown by times; deduced from the time of maximum of the first periodical term in the analysis of individual tides.*

Taking the difference of the means of large intervals and small intervals in the last Table, we deduce from them the value of $\frac{S}{M}$ by the same formula as that employed in page 42. The results are as follows:—

Station.	Difference of means of large intervals and small intervals.	Value of $\frac{S}{M}$	Station.	Difference of means of large intervals and small intervals.	Value of $\frac{S}{M}$
	m			m	
Kilbaha	58	0·37	Ballycastle	62	0·40
Kilrush	59	0·38	Glenarm	50	0·33
Foynes Island....	55	0·36	Donaghadee	54	0·35
Limerick.....	52	0·34	Ardglass.....	58	0·37
Casleh Bay.....	60	0·39	Clogher Head ..	57	0·37
Galway	59	0·38	Kingstown	56	0·36
Old Head	56	0·36	Dunmore East ..	58	0·37
Mullaghmore	62	0·40	New Ross	54	0·35
Buncrana	66	0·42	Passage West....	55	0·36
Port Rush	74	0·48	Castle Townsend..	58	0·37
Carrowkeel.....	64	0·41			

These are the true values of $\frac{S}{M}$ *, supposing that the process for finding them has been correctly followed. I omit here all deductions as to the absolute maximum of semimenstrual inequality in time at each station, for a reason that will be explained in Section XV.

To obtain the apparent age of tide as shown by times, a process was used analogous to that in page 43 or that in page 106. The times were ascertained (by interpolating between the times in the Tables of formulæ in Section X.) at which the actual interval of analysed high water from moon's transit two days previous coincided with the mean interval in the Table of page 107. These times were then compared with the times at which the moon's hour-angle from the sun was 0^h, 6^h, 12^h, 18^h; namely, June 22, 9^h, June 30, 20^h, July 7, 19^h, July 14, 21^h, July 21, 23^h, July 30, 16^h, August 6, 4^h, August 13, 5^h, August 20, 16^h. The difference was considered to be the apparent age of the tide as given by the times. The following are the results:—

Station.	Apparent age of tide from times.		Station.	Apparent age of tide from times.		Station.	Apparent age of tide from times.	
	d	h		d	h		d	h
Kilbaha	1	7	Mullaghmore	0	23	Ardglass.....	1	11
Kilrush	1	10	Buncrana	1	4	Clogher Head....	1	11
Foynes Island....	1	22	Port Rush	0	9	Kingstown	1	3
Limerick.....	2	7	Carrowkeel.....	+1	0	Dunmore East ..	2	7
Casleh Bay.....	1	4	Ballycastle	-0	14	New Ross	2	18
Galway	1	3	Glenarm	+1	12	Passage West....	1	23
Old Head	1	6	Donaghadee	1	13	Castle Townsend..	1	11

* Tides and Waves, Art. 538.

The apparent age from times is at every littoral station, except Dunmore, considerably less than the age from range. At Port Rush the apparent age is small; and at Ballycastle, the diminution proceeds so far as to change the sign of the apparent age. I cannot entirely explain this difference. It indicates that large tides arrive earlier (with reference to the hour-angles of the sun and moon) than small ones; but I know not why this should happen.

Section XV.—*Comparison of the results as to mean height, range, semimenstrual inequality in height, age of tide obtained from height, establishment, semimenstrual inequality in time, and age of tide obtained from times, deduced from high and low waters only, in Sections V., VI., VII., VIII., with those deduced from the analysis of individual tides in Sections XI., XII., XIII., XIV.*

With regard to the mean heights, we have to compare the results in Section V. with those in Section XI. And first for the mean height on the whole series of observations. The stations which appear best adapted to enable us to decide on the adoption of Mean Heights or Apparent Mean Heights as our standard (that is, as most nearly related to the height of water unaffected by tides) are those upon the Shannon. And these leave no doubt that the Mean Heights (deduced from the analysis) ought to be adopted. In a current river, it is inconceivable that the height at a lower station (as Foynes Island) should be equal to that at the highest station (Limerick), as it would be if we relied on apparent mean heights. These are the only stations which throw much light on this subject, for at Dunmore East and New Ross (river stations) the two results agree; and, at the littoral stations, there is on the whole no difference of a critical kind. At the two quasi-river-stations of Bunrana and Carrowkeel, we have on both systems discordant results, one giving a mean level higher and the other lower than that of the more exposed stations. The extreme diminution of the range of tide between Port Rush and Glenarm causes no sensible alteration of the mean level, in either system of results. The greater elevation of the mean level at the northern part of the island is equally well-marked in both. For the variation of mean height under different circumstances of large and small tides, large and small declinations of the moon, and increasing and decreasing declinations of the moon, the comparison of the numbers on the two systems gives little subject for remark, except that the difference between large and small tides in the Shannon stations seems to be more strongly marked in the mean heights than in the apparent mean heights. This however does not well accord with the idea of a standard height. It will be remarked that at Limerick the difference between the mean height and the apparent mean height is nearly a foot.

With regard to the range of the tide, we have to compare the "Mean Range" in the 8th column of the Table in page 35 with the double of "Mean or M" in the 5th column of the Table on page 106. Neglecting the differences of 0.01, we may assert that, at all the stations except Mullaghmore and Dunmore East, the apparent range

is greater than $2 C_1$ in the formulæ of Section X.; and that in the river-stations the difference is considerable. Without accounting for the two exceptions, I may remark that this shows that the departure from the pure form of tide depending on a single sine is, at all the other stations, similar in some important points of its character to that in a river tide. The nature of the tide at each station will be examined more accurately in Section XVI.

With regard to the semimenstrual inequality in height and the apparent value of $\frac{S}{M}$, we have to compare the two last columns of the Table on page 105 with the two similar columns of the Table on page 35. The numbers are so nearly equal that we cannot assert that there is any certain difference. The large value of $\frac{S}{M}$ at Port Rush, the small value at Glenarm, and the general smallness from Donaghadee to Castle Townsend (like that in the river station Limerick) are equally marked in both.

With regard to the age of the tide as obtained from heights or ranges (which, as I have stated before, is the true age of the tide), we must compare the results on page 105 with those on page 38. They agree, on the whole, very well; though the ages deduced from the analysis appear to be somewhat smaller than those deduced from high and low waters. The ages deduced from the analysis also agree better among themselves. The diminution however from Port Rush to Ballycastle is remarkable. The general result seems to be that on the south-western coast of Ireland the age of the tide is about one day twenty hours.

For the establishment, we must compare the Table of page 106 with that of page 39. Only at Limerick and New Ross is the difference considerable: at these stations it amounts to about twenty minutes.

For the semimenstrual inequality in time, we must compare the numbers in page 107 with those of page 42. And here it will at once be remarked that a great and important change has been made in the resulting values of $\frac{S}{M}$ by the new mode of treating them. The values at Old Head and Glenarm and the following stations are increased, and those at Port Rush and Ballycastle are diminished, till they agree sufficiently well with the others. This change arises from two causes. First, the determination of times from the analysis is vastly more accurate than that from the estimation of the times of high and low water. Secondly, in Section VIII. the groups for large intervals and small intervals were divided at each station from a consideration of the magnitudes of the intervals themselves, whereas in Section XIV. they were divided from consideration of a totally different circumstance, namely the age of the tide as shown by the time of occurrence of the mean value of range. Strange as it may appear, the former method was incorrect, and the latter is correct. The former method is affected by a circumstance which ought not to enter into the formation of this result at all, namely the change in the time of station-tide, not depending on the change in the time of sea-tide, but depending on the change which the character of the tidal for-

mula in Section X. undergoes when the magnitude of the range is altered. Thus, the maximum and minimum intervals of sea-tide (which are the objects of our search) occur at the times (corrected for age of tide) when the moon's hour-angle from the sun is 3^h , 9^h , &c. But from hour-angle 9^h to hour-angle 12^h the range of tide is increasing: the modification of time of station-tide as related to time of sea-tide is therefore increasing; a second inequality of time is therefore combined with the first, having a different time of vanishing: the time of vanishing of the compound inequality is therefore different from that of the first inequality, and the maximum magnitude of the compound inequality is different from that of the first. And this produces its full effect if we make our divisions of groups with reference to the time of vanishing of the compound inequality. But if we make our divisions strictly at the times when the first inequality ought to vanish, then, though every individual time be affected by the second inequality, yet there are the same number of instances affected in the same way on opposite sides of our places of division, and their effect disappears in the final result. But as the full compound effect is not used for our final result, conversely we cannot from our final result infer the maximum magnitude of the full compound effect, or the maximum value of the semimenstrual inequality in time.

These considerations appear to be deserving of the utmost attention in investigating the most important single result which can be deduced from the tides, namely the proportion of the hydrodynamical effects of the sun and the moon.

The mean of all the values of $\frac{S}{M}$ in page 108 is 0.38. It is probable that, on attending more scrupulously to the age of the tide at the different stations, results would have been found agreeing more closely with each other: but I think it likely that the mean would scarcely have been altered.

With regard to the apparent age of tide obtained from times, we must compare the numbers in page 108 with those in page 44. The general result of the comparison is, that the mean of the opposite ages deduced from high and low water agrees with that deduced from the analysis as nearly as can be expected (where a small error in the estimated time of high or low water would produce a great effect on the resulting age). They agree in giving a small apparent age at Port Rush and Ballycastle. The ages deduced from range agree also in this. Now this result* is exactly that which would follow from the supposition that a tidal wave with a large range travels more quickly over the shallow bottom than one with a small range: and that this holds not for the phase of high water or low water only, but for the zero of the angle p . Or it would follow from the supposition that a wave of short period travels more quickly than one of long period (the tidal day near conjunction being shorter than that near quadrature). Neither of these points has been established by theory; but the former appears to be very probable.

* Tides and Waves, Art. 463.

Section XVI.—*Remarks on the succeeding terms of the expressions for individual tides, as related to the magnitude of the tide, to the position on the sea-coast, to the position on the river, &c.; comparison with the terms given by the theory of waves; discussion of the quarto-diurnal tide.*

In order to reduce to a smaller number the numerous formulæ of Section X., and to render the relations of their coefficients and arguments at once accurate and distinct, the formulæ have been divided into groups corresponding to large tides and small tides, the times of division being the same as those used in the discussions of the semimenstrual inequality of height. Then, for the large tides, the numbers $\frac{32}{\pi} A_2$ have been collected and their mean has been taken; similarly, the mean of the numbers $\frac{32}{\pi} B_2$ has been taken; and from these, by the treatment described in Section X., a term similar to $C_2 \sin(2 \text{ phase} + \phi_2)$ is formed. The mean of all the angles ϕ_1 is also taken, and its double is subtracted from the number corresponding to ϕ_2 . A similar process is used for 3 phase and 4 phase. As for some tides ϕ_1 is nearly equal to 90° and for others is nearly equal to 270° ; the expressions in the latter case are made to admit of combination with those in the former case by subtracting 180° from ϕ_1 and by changing the signs of A_3 and B_3 . The small tides are treated in the same manner. The following Table contains the result.

Variable part of the formulæ for the elevation of the water at each of the Stations in
Large Tides and Small Tides.

Kilbaha	{ Large tides. Small tides.	ft. 6.12 .sin p 3.73 .sin p	ft. + 0.16 .sin (2p + 164) + 0.08 .sin (2p + 208)	ft. + 0.03 .sin (3p + 231) + 0.06 .sin (3p + 248)	ft. + 0.03 .sin (4p + 88°) + 0.01 .sin (4p + 120)
Kilrush	{ Large tides. Small tides.	6.54 .sin p 4.19 .sin p	+ 0.19 .sin (2p + 104) + 0.12 .sin (2p + 128)	+ 0.06 .sin (3p + 195) + 0.06 .sin (3p + 210)	+ 0.03 .sin (4p + 167) + 0.02 .sin (4p + 175)
Foynes Island	{ Large tides. Small tides.	7.35 .sin p 4.96 .sin p	+ 0.55 .sin (2p + 111) + 0.37 .sin (2p + 124)	+ 0.26 .sin (3p + 162) + 0.11 .sin (3p + 175)	+ 0.03 .sin (4p + 277) + 0.01 .sin (4p + 327)
Limerick	{ Large tides. Small tides.	8.98 .sin p 6.21 .sin p	+ 0.90 .sin (2p + 75) + 0.74 .sin (2p + 111)	+ 0.68 .sin (3p + 125) + 0.40 .sin (3p + 145)	+ 0.38 .sin (4p + 160) + 0.15 .sin (4p + 182)
Casleh Bay	{ Large tides. Small tides.	6.35 .sin p 3.83 .sin p	+ 0.16 .sin (2p + 165) + 0.07 .sin (2p + 219)	+ 0.12 .sin (3p + 214) + 0.07 .sin (3p + 238)	+ 0.06 .sin (4p + 348) + 0.03 .sin (4p + 35)
Galway	{ Large tides. Small tides.	6.41 .sin p 3.89 .sin p	+ 0.18 .sin (2p + 157) + 0.08 .sin (2p + 204)	+ 0.18 .sin (3p + 204) + 0.12 .sin (3p + 233)	+ 0.16 .sin (4p + 349) + 0.04 .sin (4p + 38)
Old Head	{ Large tides. Small tides.	5.59 .sin p 3.43 .sin p	+ 0.08 .sin (2p + 154) + 0.04 .sin (2p + 216)	+ 0.04 .sin (3p + 196) + 0.10 .sin (3p + 257)	+ 0.04 .sin (4p + 282) + 0.01 .sin (4p + 237)
Mullaghmore	{ Large tides. Small tides.	5.35 .sin p 3.25 .sin p	+ 0.06 .sin (2p + 160) + 0.04 .sin (2p + 217)	+ 0.07 .sin (3p + 14) + 0.03 .sin (3p + 134)	+ 0.06 .sin (4p + 184) + 0.02 .sin (4p + 254)
Buncrana	{ Large tides. Small tides.	5.84 .sin p 3.37 .sin p	+ 0.14 .sin (2p + 165) + 0.04 .sin (2p + 199)	+ 0.08 .sin (3p + 262) + 0.04 .sin (3p + 256)	+ 0.04 .sin (4p + 295) + 0.01 .sin (4p + 237)
Port Rush	{ Large tides. Small tides.	2.56 .sin p 1.38 .sin p	+ 0.08 .sin (2p + 153) + 0.04 .sin (2p + 220)	+ 0.12 .sin (3p + 257) + 0.07 .sin (3p + 287)	+ 0.01 .sin (4p + 207) + 0.02 .sin (4p + 172)
Carrowkeel	{ Large tides. Small tides.	3.35 .sin p 1.95 .sin p	+ 0.07 .sin (2p + 238) + 0.01 .sin (2p + 319)	+ 0.11 .sin (3p + 205) + 0.06 .sin (3p + 199)	+ 0.01 .sin (4p + 152)
Ballycastle	{ Large tides. Small tides.	1.42 .sin p 0.88 .sin p	+ 0.07 .sin (2p + 308) + 0.06 .sin (2p + 338)	+ 0.07 .sin (3p + 276) + 0.03 .sin (3p + 311)	+ 0.03 .sin (4p + 218) + 0.01 .sin (4p + 293)
Glenarm	{ Large tides. Small tides.	2.89 .sin p 2.47 .sin p	+ 0.17 .sin (2p + 76) + 0.05 .sin (2p + 50)	+ 0.07 .sin (3p + 227) + 0.04 .sin (3p + 237)	+ 0.01 .sin (4p + 310)
Donaghadee	{ Large tides. Small tides.	5.24 .sin p 3.96 .sin p	+ 0.07 .sin (2p + 56) + 0.03 .sin (2p + 318)	+ 0.03 .sin (3p + 174) + 0.03 .sin (3p + 230)	+ 0.01 .sin (4p + 327) + 0.01 .sin (4p + 85)
Ardglass	{ Large tides. Small tides.	6.90 .sin p 4.96 .sin p	+ 0.18 .sin (2p + 98) + 0.07 .sin (2p + 117)	+ 0.04 .sin (3p + 145) + 0.04 .sin (3p + 217)	+ 0.03 .sin (4p + 125) + 0.03 .sin (4p + 161)
Clogher Head	{ Large tides. Small tides.	6.79 .sin p 4.81 .sin p	+ 0.26 .sin (2p + 109) + 0.13 .sin (2p + 143)	+ 0.08 .sin (3p + 166) + 0.07 .sin (3p + 180)	+ 0.01 .sin (4p + 233) + 0.02 .sin (4p + 241)
Kingstown	{ Large tides. Small tides.	5.26 .sin p 3.69 .sin p	+ 0.38 .sin (2p + 143) + 0.23 .sin (2p + 165)	+ 0.02 .sin (3p + 7) + 0.01 .sin (3p + 225)	+ 0.02 .sin (4p + 118) + 0.02 .sin (4p + 171)
Dunmore East	{ Large tides. Small tides.	5.73 .sin p 3.91 .sin p	+ 0.14 .sin (2p + 12) + 0.11 .sin (2p + 3)	+ 0.15 .sin (3p + 94) + 0.08 .sin (3p + 103)	+ 0.01 .sin (4p + 288) + 0.01 .sin (4p + 198)
New Ross	{ Large tides. Small tides.	6.16 .sin p 4.51 .sin p	+ 0.47 .sin (2p + 348) + 0.30 .sin (2p + 346)	+ 0.26 .sin (3p + 101) + 0.16 .sin (3p + 95)	+ 0.03 .sin (4p + 116) + 0.01 .sin (4p + 63)
Passage West	{ Large tides. Small tides.	5.94 .sin p 4.19 .sin p	+ 0.29 .sin (2p + 298) + 0.24 .sin (2p + 316)	+ 0.09 .sin (3p + 87) + 0.07 .sin (3p + 95)	+ 0.04 .sin (4p + 73) + 0.02 .sin (4p + 43)
Castle Townsend	{ Large tides. Small tides.	4.79 .sin p 3.33 .sin p	+ 0.21 .sin (2p + 297) + 0.16 .sin (2p + 310)	+ 0.04 .sin (3p + 259) + 0.03 .sin (3p + 302)	+ 0.02 .sin (4p + 331) + 0.03 .sin (4p + 73)

In order to institute a comparison of these numerical results with the formulæ given by theory, it may be convenient to premise the following expressions:—

If the tide were created in a uniform channel, forming a great circle round the earth, the expression for the height of the water would have the form

$$A \cdot \sin p + B \cdot \sin \overline{2p + 90^\circ},$$

where B would have the same sign as A if the velocity of the tide-wave were greater than $\sqrt{\frac{gk}{4}}$, k being the depth of the channel.

If the tide at the mouth of a gulf be a pure tide, or one in which the elevation is expressed by $a \cdot \sin p$, then the elevation at any point in the bay will be expressed by the formula

$$A \cdot \sin p + B \cdot \sin \overline{2p + 90^\circ},$$

where (in the case of a gulf sufficiently long to have a tidal node) A and B have different signs from the mouth of the gulf to the node, and afterwards have similar signs.

If the tide at the mouth of an indefinitely long river be a pure tide, then the elevation at any point in the river will be expressed by the formula

$$A \cdot \sin p + B \cdot \sin 2p,$$

where B has the same sign as A if the section of the river be a parallelogram, but may have the opposite sign if the section expand very much at the top.

In all cases $\frac{B}{A}$ is a quantity of the same order as $\frac{\text{vertical oscillation of the water}}{\text{depth of the water}}$.

I shall now proceed to examine the results deducible from the last Table.

The stations Kilbaha, Casleh Bay, Old Head, Mullaghmore, may fairly be considered as littoral stations on the open Atlantic ocean. And their formulæ agree among themselves almost absolutely to the second term, and in a great measure to the third term. They show clearly that the Atlantic tide there is not a pure tide. But the form of the argument does not agree with either of the two first formulæ just cited, which alone can apply to it. At Castle Townsend, which is nearly as much exposed, but on a different side of the island, the formula agrees pretty well with the first of those above, supposing the depth of the sea very great.

At Dunmore East the tide has nearly assumed the form of a river tide.

Proceeding from Kingstown (where the character of the tide is similar to that at Kilbaha, &c.) to Clogher Head, Ardglass, and Donaghadee, the argument of the second term undergoes a progressive change, its phase being less advanced. As the epoch of the first term is absolutely the same at these stations (the tide being simultaneous at all of them), it appears that the wave represented by the second term is progressive. It seems therefore that it does not originate in the peculiarities of a gulf-tide (contained in the second formula just cited), but that it has been created either on the open sea or in the shallower water between Ireland and Cornwall, and now travels on as an independent wave.

From Mullaghmore to Port Rush, the second wave appears to travel with the same speed as the principal wave. In passing through the narrow channel to Glenarm, its phases appear to increase much more rapidly than those of the principal wave; or, it appears to travel more slowly, or in the opposite direction. This however is probably only an instance of the forced progression of the phases of wave in the channel connecting two tidal seas.

The general relations however of the waves depending on $2p$ at the different littoral stations will be seen more clearly from the following process:—Take the establishment of each station from Section XIII., and convert it into degrees at the rate of 1440° for a tidal day, and subtract the angle thus found from the angle added to $2p$. It is evident now that our phase p' at every station is referred to the same origin, namely to the time of the moon's transit at Greenwich. The expressions thus obtained for the quarto-diurnal waves are the following:—

Station.	Large tides.	Small tides.	Station.	Large tides.	Small tides.
Kilbaha	ft. $0\cdot16 \cdot \sin(2p' + 244)$	ft. $0\cdot08 \cdot \sin(2p' + 295)$	Donaghadee . .	ft. $0\cdot07 \cdot \sin(2p' + 122)$	ft. $0\cdot03 \cdot \sin(2p' + 41)$
Casleh Bay . .	$0\cdot16 \cdot \sin(2p' + 235)$	$0\cdot07 \cdot \sin(2p' + 296)$	Ardglass	$0\cdot18 \cdot \sin(2p' + 163)$	$0\cdot07 \cdot \sin(2p' + 199)$
Galway	$0\cdot18 \cdot \sin(2p' + 223)$	$0\cdot08 \cdot \sin(2p' + 277)$	Clogher Head . .	$0\cdot26 \cdot \sin(2p' + 167)$	$0\cdot13 \cdot \sin(2p' + 218)$
Old Head . . .	$0\cdot08 \cdot \sin(2p' + 208)$	$0\cdot04 \cdot \sin(2p' + 278)$	Kingstown	$0\cdot38 \cdot \sin(2p' + 195)$	$0\cdot23 \cdot \sin(2p' + 235)$
Mullaghmore .	$0\cdot06 \cdot \sin(2p' + 198)$	$0\cdot04 \cdot \sin(2p' + 263)$	Dunmore East . .	$0\cdot14 \cdot \sin(2p' + 72)$	$0\cdot11 \cdot \sin(2p' + 71)$
Port Rush . . .	$0\cdot08 \cdot \sin(2p' + 113)$	$0\cdot04 \cdot \sin(2p' + 190)$	Passage West . .	$0\cdot29 \cdot \sin(2p' + 344)$	$0\cdot24 \cdot \sin(2p' + 10)$
Ballycastle . .	$0\cdot07 \cdot \sin(2p' + 203)$	$0\cdot06 \cdot \sin(2p' + 245)$	Castle Townsend	$0\cdot21 \cdot \sin(2p' + 7)$	$0\cdot16 \cdot \sin(2p' + 27)$
Glenarm	$0\cdot17 \cdot \sin(2p' + 156)$	$0\cdot05 \cdot \sin(2p' + 147)$			

The expressions for Courtown, the station intermediate between Kingstown and Dunmore East, are (as I remark by anticipation from the next section) intermediate between those for Kingstown and Dunmore East, but nearly coinciding with the former.

The variations in the values of the constants attached to $2p$ in the Table of page 113, seem to make it impossible for us to attribute this term to the local circumstances of each port, and the consideration of the Courtown tides in the next section will confirm this. The order of the numbers attached to $2p'$ in the last Table shows that it may be considered as a progressive wave, beginning at Kingstown nearly, and travelling both ways round the coast as far as Donaghadee. But whether such a thing is mechanically possible, or whether it can be true that the quarto-diurnal wave (which necessarily is created by the semidiurnal wave flowing over the shallower seas between Ireland and Cornwall) can show itself as a great swell opposite Kingstown, and can then be propagated even opposite to the semidiurnal wave and round the island, are points which I cannot explain.

On the whole, I am not able to pronounce with any confidence on the origin of this wave, but I have no doubt that, having been created, it travels along independently, and therefore that its existence is not due to the local circumstances of the several stations.

In regard to the river stations, I may remark that the Shannon does, in consequence of the barriers to the tide at Limerick, resemble a gulf in its tidal character; and the second of the formulæ above ought therefore to apply to it; and, as will easily be seen, it does apply with considerable approximation. The river Barrow, upon which New Ross is situated, is not obstructed in the same manner, and therefore we might expect the third formula to apply, and it does apply very nearly.

I omit discussion of the third and fourth terms, because theory, in a shape applicable to cases of nature, has not yet been extended sufficiently far. I may however observe that, as I have shown with regard to the tide at Deptford*, and to those at Southampton and Ipswich†, so also at Limerick, and in some measure at Foynes Island and New Ross, the third term is almost as important as the second, and the fourth is one of considerable magnitude.

I may also call the attention of the wave-theorist to this circumstance, that the difference between the coefficients for large tides and for small tides does not appear sufficiently great in relation to the difference between the coefficients of the first term for large tides and small tides. The coefficient of the first term being considered as of the first order, that of the second term would consist of a series whose leading term was of the second order, &c. The departure from the proportions given by this consideration may depend upon the succeeding terms of the series.

It is also to be remarked that there is an undoubted difference between the arguments for large tides and for small tides. This seems to show that each sine is accompanied by a cosine, and that their coefficients have, for their leading terms, terms of different orders in respect of the first coefficient of semi-range.

Section XVII.—*Separate discussion of the tidal observations made at Courtown.*

The observers at Courtown soon discovered that it was impossible to adhere to the instructions sketched in Section I. The tide was sometimes apparent as a semi-diurnal tide, but with considerable irregularity; at other times, the character of semidiurnal tide was (to common observation) completely lost, and in its stead there was a small tide four times a day; in all cases the tide was small. In this state of things, the course which they adopted was, to observe continuously whenever the semidiurnal tide was not distinctly marked, and to follow the usual rule (with some extension of observations) when it was well-marked. In this manner a tolerably complete and very important set of observations has been secured. In several cases the observations have been interrupted by the discharge of water from the sluices for scouring the harbour of Courtown; in some instances there has been no difficulty in filling up the observations by conjecture, in others I have been obliged to adopt the limits of the tide (in the form of analysis) to these interruptions.

In order to apply the method of analysis explained in Section X., it was necessary to fix upon precise limits for each tide. But as no limits could be obtained at Cour-

* Philosophical Transactions, 1842, p. 4.

† Ibid. 1843, pp. 49 and 53.

town (as at other stations) from the observations themselves, it was necessary to take them from another station. The station selected for this purpose was Ardglass. The limits and the divisions into twelfths and sixteenths for Ardglass were therefore adopted for Courtown, as far as the continuity of observations permitted. Where (as is just mentioned) it was necessary to change the limits, this was done if possible by altering the limits and all the divisions by three or six of the twelfths (corresponding to four or eight of the sixteenths); in other cases they were all altered by a definite time. Then the means of the heights in these observations were treated in the usual way.

A correction for diurnal tide was indispensable (the diurnal tide being, at some times, as large as the semidiurnal). For obtaining this from the observations there were two means. One was, by means of the investigations connected with tertio-diurnal tide to be detailed in the next section. These gave the diurnal tide for the beginning and the fourth parts of each of the whole day's group used there. These were the diurnal coefficients proper to be used in the semidiurnal groups composing each day's group. But the corresponding diurnal coefficient applicable at the times of any Ardglass high or low water was easily deduced from them by taking the sum of the products of the coefficients next it by the cosines of their respective distances from it (considering 360° as corresponding to a tidal day). Another method was, to select from the observed heights those which corresponded to the times of Ardglass high water and Ardglass low water, and to treat them by the method of fourth differences explained in Section III.; as these heights ought (in relation to each other) to be perfectly free from the effect of semidiurnal tide and of all tides occurring at portions of a semidiurnal tide. Using then these two methods, and adopting the mean of their results when both could be applied, a number of diurnal coefficients were obtained from the observations themselves. On comparing these with the diurnal coefficients at the neighbouring stations, it was found that the coefficients at Courtown might very well be represented by the mean of those for Kingstown and Dunmore East at the same time. Accordingly, for all the times for which no diurnal tide could be safely extracted from the observations, the mean of coefficients for Kingstown and Dunmore East was used; and from these, when necessary, the coefficients for other times were deduced by the operation described above. The process then pursued was exactly the same as in other cases, except that no correction was attempted for rise of water. The results are the following, which differ in form from preceding results only in this circumstance, that the origin of phase is the time of mean water at Ardglass preceding high water, and that therefore an angle expressed by a number of degrees must be added to the phase to form the argument of the first variable term.

Height of the water in each individual tide at Courtown, excluding diurnal tide, where the origin of p is at the same time as at Ardglass (p. 82 and 83),

Analysed time of high water, corresponding to $p+c_1=90$.			A_0 .	C_1 .	c_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .	
1842.	h	m	ft.	ft.		ft.		ft.		ft.		
June	22.	20	13	16.93	0.74	109	0.43	170	0.03	320	0.02	103
	23.	8	30									
	23.	20	46	17.30	0.74	91	0.36	133	0.05	93	0.02	265
	24.	9	25									
	24.	22	4	17.55	0.50	73	0.36	102	0.04	180	0.04	20
	25.	10	11									
	25.	22	18	17.03	0.78	83	0.33	127	0.08	253	0.05	337
	26.	10	13									
	26.	22	9	16.55	0.46	107	0.33	122	0.03	178	0.04	200
	27.	10	21									
	27.	22	5	16.58	0.38	125	0.29	128	0.04	160	0.04	152
	28.	10	30									
	28.	22	39	16.66	0.17	133	0.22	130	0.03	238	0.03	298
	29.	11	24	16.64	0.25	112	0.05	130	0.05	106	0.01	217
	29.	22	42	16.71	0.17	199	0.24	130	0.03	275	0.02	314
	30.	9	29	16.59	0.18	199	0.28	148	0.03	164	0.03	60
	30.	21	3	16.69	0.36	251	0.19	126	0.02	59	0.03	237
July	1.	8	50	16.65	0.23	239	0.19	138	0.04	164	0.04	227
	1.	21	14	16.66	0.28	256	0.23	128	0.03	280	0.03	333
	2.	9	21									
	2.	16	7	16.88	0.09	58	0.18	83	0.03	296	0.05	137
	3.	4	42									
	3.	17	26	17.59	0.13	51	0.15	106	0.04	127	0.05	200
	4.	5	51									
	4.	22	34	17.40	0.12	291	0.21	143	0.12	114	0.04	358
	5.	11	4									
	5.	18	38	16.82	0.51	74	0.29	134	0.05	50	0.04	350
	6.	7	8									
	6.	19	52	17.21	0.37	63	0.52	120	0.11	207	0.05	43
	7.	8	12									

	8.	20	42	16.65	1.29	81	0.44	132	0.10	138	0.05	260
	9.	9	12									
	9.	21	32	16.62	1.57	82	0.37	134	0.11	87	0.07	231
	10.	9	58									
	10.	22	25	17.62	1.18	77	0.16	96	0.09	101	0.08	66
	11.	10	40									
	11.	22	56	16.48	1.26	82	0.34	116	0.10	100	0.06	202
	12.	11	24									
	12.	23	51	16.59	0.93	86	0.42	127	0.01	1	0.03	199
	(13.	9	59)
	(13.	20	7)
	(14.	6	15)
	(14.	16	23)
	15.	2	31	16.08	0.27	58	0.43	121	0.04	282	0.07	345
	15.	17	56	16.58	0.04	339	0.24	136	0.06	329	0.02	192
	16.	6	26									
	16.	20	45	17.05	0.26	288	0.33	106	0.08	101	0.11	178
	17.	9	36	17.11	0.09	278	0.15	150	0.11	328	0.10	358
	17.	22	4	16.89	0.10	283	0.38	135	0.01	160	0.03	164
	18.	2	39									
	18.	15	45	16.90	0.01	155	0.35	115	0.17	106	0.08	231
	19.	5	18	16.81	0.18	130	0.37	142	0.10	40	0.12	113
	19.	19	5	16.82	0.17	93	0.29	115	0.16	304	0.06	304
	20.	7	58	16.70	0.21	53	0.35	125	0.14	122	0.10	131
	20.	19	41	16.80	0.58	68	0.22	151	0.04	294	0.03	341
	21.	7	35	16.61	0.57	93	0.36	138	0.05	169	0.05	233
	21.	20	2	16.47	0.74	91	0.28	141	0.04	131	0.01	12
	22.	8	22									

expressed by the formula $A_0 + C_1 \sin(p + c_1) + C_2 \sin(2p + c_2) + C_3 \sin(3p + c_3) + C_4 \sin(4p + c_4)$, and p increases by 360° during one complete tide at Ardglass.

Analysed time of high water, corresponding to $p + c_1 = 90^\circ$.		A_0 .	C_1 .	c_1 .	C_2 .	c_2 .	C_3 .	c_3 .	C_4 .	c_4 .
1842.	h m	ft.	ft.	°	ft.	°	ft.	°	ft.	°
July	22. 20 40	16.32	0.80	89	0.34	135	0.05	152	0.01	158
	23. 8 54									
	23. 21 9	16.32	0.89	96	0.37	145	0.06	79	0.07	158
	24. 9 18									
	24. 21 28	16.51	0.74	103	0.23	125	0.12	211	0.05	198
	25. 9 41									
	25. 21 54	16.37	0.73	105	0.35	128	0.07	248	0.02	270
	26. 10 17	16.36	0.80	102	0.21	155	0.12	106	0.02	75
	26. 22 25	16.41	0.60	111	0.27	124	0.06	168
	27. 10 35	16.34	0.61	106	0.25	129	0.04	130
	27. 23 13	16.42	0.39	108	0.30	117	0.04	155	0.03	341
	28. 11 7	16.37	0.35	109	0.10	127	0.06	171	0.04	161
	28. 22 37	16.44	0.32	158	0.23	128	0.07	148	0.01	222
	29. 10 2	16.36	0.24	163	0.22	132	0.04	254	0.05	341
	29. 21 25	16.38	0.25	208	0.24	109	0.02	61	0.03	43
	30. 8 42	16.31	0.25	233	0.24	134	0.05	10	0.05	0
	30. 20 35	16.25	0.41	247	0.18	116	0.01	105	0.03	245
	31. 8 38	16.21	0.43	259	0.19	150	0.01	249	0.02	18
	31. 20 55									
August	1. 9 8	16.37	0.48	266	0.27	118	0.02	87	0.02	107
	1. 21 6	16.53	0.34	287	0.21	148	0.02	46	0.02	36
	2. 9 20	16.57	0.36	305	0.25	149	0.01	226	0.02	78
	2. 19 45	16.42	0.09	9	0.26	150	0.05	30	0.03	122
	3. 7 36	16.32	0.22	31	0.31	159	0.02	63	0.02	39
	3. 18 27	16.48	0.35	83	0.20	148	0.06	110	0.03	177
	4. 6 44	16.47	0.60	82	0.26	170	0.09	45	0.09	131
	4. 19 17	16.76	0.88	79	0.35	152	0.04	274	0.07	27
	5. 8 1	16.73	1.13	69	0.41	156	0.04	216	0.05	80
	5. 20 0	16.58	1.23	80	0.33	140	0.09	53	0.08	55
	6. 8 18									
	6. 20 36	16.65	1.53	78	0.41	110	0.11	128	0.05	3
	7. 9 5									
	7. 21 34	16.46	1.89	75	0.45	141	0.03	169	0.06	127
	8. 9 56									
	8. 22 18	16.64	1.70	74	0.52	127	0.07	118	0.01	225
	9. 10 4									
	9. 21 50	17.31	1.60	114	0.28	225	0.21	48	0.09	226
	10. 10 45									
	10. 23 40	16.56	1.32	82	0.33	127	0.11	269	0.09	215
	11. 11 57									
	12. 0 14	16.62	0.59	88	0.41	142	0.02	329	0.03	141
	12. 13 15	16.63	0.58	72	0.22	120	0.05	222	0.03	59
	12. 23 49	16.26	0.09	125	0.29	120	0.06	356
	13. 9 26	16.29	0.11	209	0.31	123	0.11	29	0.01	247
	13. 20 54	16.20	0.25	274	0.27	117	0.07	88	0.11	187
	14. 8 41	16.29	0.34	260	0.39	150	0.05	354	0.01	218
	14. 21 8	16.32	0.34	287	0.35	126	0.06	243	0.03	235
	15. 9 33	16.35	0.39	277	0.21	130	0.04	27	0.02	208
	15. 21 50	16.55	0.13	307	0.17	148	0.03	157	0.02	350
	16. 10 6	16.31	0.25	300	0.24	113	0.08	94	0.05	178
	16. 22 50									
	17. 6 30	16.78	0.21	78	0.14	122	0.05	169	0.02	47
	17. 18 41	16.58	0.19	84	0.28	120	0.07	250	0.08	325
	18. 7 24									
	18. 20 7	17.07	0.17	64	0.30	122	0.05	76	0.04	90

	20. 20 20	16.57	0.92	89	0.23	123	0.04	124	0.02	267

Upon inspecting the numbers in the column headed c_1 , it will be perfectly evident that there is an error on August 9. The semidiurnal tide on that day is (comparatively) large, its whole range exceeding three feet; and there is no instance throughout all the observations of an irregularity equal to that which corresponds to an anomaly of 30° with that range. I have no doubt that all the observations are recorded too early by one hour, an error which would easily be committed at beginning, and which, where the observations are entered on forms ready prepared, would be retained to the end. Correcting on this supposition, the three successive times of high water would be August 9^d 10^h 34^m, August 9^d 22^h 50^m, and August 10^d 11^h 15^m; and the values of c_1 , c_2 , c_3 , c_4 , for August 9^d 22^h 50^m would be nearly 84° , 165° , 218° , and 106° .

Next I would remark that there is undoubtedly an error of the same kind in the tide of July 4–5; but as the tide is then very small, I have not ventured to state precisely the alteration which I would propose.

Thirdly, observing that where the tide is very small, the hours on successive days occur earlier, in each of the instances where the order is well-marked (as from June 27 to July 3, from July 27 to July 31, and from August 12 to August 14), there can be no doubt that the same thing must hold during the interruption of observations about July 13 and 14; and thus it will be seen that there are certainly four high waters lost at that time. I have inserted four numbers by simple interpolation, to show, within two or three hours, the times of the lost high waters.

It is also to be remembered that nothing can be inferred from such tides as those of July 15–16 and July 18, where the coefficients are 0.04 and 0.01.

Bearing these remarks in mind, and giving particular attention to the second half of the observations, which, both for the regularity of the system pursued by the observers and for the agreement of the results, is greatly superior to the first, we arrive at the following conclusions:—

The angle c_1 increases continually, and its increase amounts to 360° in about fourteen days. When its value is not far from 360° , its increase is extremely rapid. One of these jumps occurs between August 2^d 9^h and August 2^d 19^h, and one between August 16^d 22^h and August 17^d 6^h; one also between July 2^d 9^h and July 2^d 16^h; another takes place between July 17^d 22^h and July 19^d 5^h, but (the results being at that time somewhat irregular) the time cannot be precisely pointed out. It is evident from this that the Courtown tides are more numerous than the Ardglass tides by one tide in fourteen days nearly.

The mean solar time of high water does not increase constantly as at other stations, but oscillates backwards and forwards. Thus, from July 19 to August 18 (in which period the tides at other stations have gradually retarded by twenty-four hours), the evening tides always occur between 5^h 18^m and 13^h 15^m, and the morning tides always occur between 18^h 27^m and 24^h 14^m, each time having twice oscillated between its extreme limits in that period.

From these circumstances it is plain that the time of high water (confining our remarks to the term $C_1 \sin(p+c_1)$) respects mainly the time of the sun's transit and not that of the moon's transit; and therefore, at Courtown, *the solar tide is greater than the lunar tide*. This is, I believe, the only place on the earth in which such a result has been distinctly obtained. The observations of Sir EDWARD BELCHER* show that at Otaheite the solar tide is as nearly as possible equal to the lunar tide.

The times following lunar syzygy by two days were June 24, 9^h, July 9, 19^h, July 23, 23^h, August 8, 4^h, August 22, 16^h; and about these times the luni-solar tide is greatest at all the other stations. About these times also the soli-lunar tide at Courtown is greatest.

The solar hour of high water at Kingstown, at the highest tides, is about 12^h 30^m, that at Dunmore East is about 6^h 40^m. These are the two stations nearest to Courtown on the north and south sides. The solar hour of high water at Courtown at the same times is about 9^h 30^m. At these times the effects of the sun and the moon are simultaneous as to phase, so that we may treat the result as if there were only a single wave. It would seem then that the transition from a tide of elevation at Dunmore East to one of simultaneous depression at Kingstown, and *vice versa*, is not effected entirely by a node dividing the elevated wave from the depressed wave. It appears that there is also a small progressive wave. The geometrical representation appears to be this; that there is a large stationary wave, having a node near Courtown, and making high water simultaneous in all parts of the inland sea or Irish Sea, and synchronic with low water in the exterior sea; and that there is mingled with it a very small progressive wave. As to the mechanical explanation of it, I can offer nothing positive. But I would suggest for the consideration of wave-theorists, whether, in the case of a gulf (as the Irish Sea) having a small outlet (like the North Channel), it be possible that the fluctuation may be represented correctly on mechanical principles by a combination of the stationary wave peculiar to a gulf with the progressive wave peculiar to the channel.

Returning now to the consideration of the magnitude of the tide, it is evident that the coefficient of the lunar tide has been diminished in a far greater degree than that of the solar tide. There is one explanation of this which is very plausible, and which I have no doubt is the true one, namely that the node for the lunar tide and the node for the solar tide do not coincide (which, on account of the difference of the periods of these tides, we should expect *à priori*), and that the node for the lunar tide is much nearer to Courtown than is the node for the solar tide. It is clearly possible that, by varying our choice of stations, we might vary the proportion of the two effects in any degree whatever. Nay, by choosing a station between the two nodes, we might have the solar and lunar effects to conspire when they are opposed at other places, and *vice versa*; and thus a station would be found where the spring tides occur at the same time as neap tides at other places. This does not occur at

* Philosophical Transactions, 1843, p. 55, &c.

Courtown; but the reader, in reflecting on this, will see the importance of our comparison of the time of the largest tides at Courtown with the time of the largest tides at the other stations.

We shall now proceed to examine the second periodical term; which will be found not less remarkable than the first.

On glancing over the values of c_2 (first correcting that on August 9 as I have suggested), the reader cannot fail to be struck with the general uniformity of the numbers. It is quite evident that this term has no respect to the sun's transit, but that it respects only the moon's transit, or the commencement of the luni-solar tide. If we look also to the coefficient C_1 , we find that its magnitude is considerable, sometimes exceeding that of the first term. It is clear therefore that this term does not originate as a derivative from the first term, produced by the local circumstances of the port. It does not change greatly, but nevertheless has on the whole larger values about the times of large tides than about the times of small tides. If we divide the observations into two groups, one corresponding to large tides and the other to small tides (the limits being the same as those for the other stations); and if we correct as before for the establishment at Ardglass (to which station the Courtown tides have been referred); and if we collect the expressions for the second periodic term at the three stations, Dunmore East, Courtown and Kingstown; we have this sequence of expressions.

	Kingstown.	Courtown.	Dunmore East.
Large tides	ft. $0\cdot38 \cdot \sin (2p' + 195^\circ)$	ft. $0\cdot35 \cdot \sin (2p' + 196^\circ)$	ft. $0\cdot14 \cdot \sin (2p' + 72^\circ)$
Small tides.	$0\cdot23 \cdot \sin (2p' + 235)$	$0\cdot26 \cdot \sin (2p' + 202)$	$0\cdot11 \cdot \sin (2p' + 71)$

It appears here quite evident that this term at Courtown is only the representation of the same quarto-diurnal tide which shows itself along the whole coast. This wave (whatever its origin may be) appears to have its greatest range and its beginning of phases at Kingstown, and to spread both ways, diminishing in range as it goes.

The succeeding terms at Courtown are insignificant.

We have now a clear representation of the apparently confused phenomena of the tides at Courtown. Both the semidiurnal tides are very much diminished, the lunar so much that its range is rather less than that of the solar tide. The quarto-diurnal tide exists in nearly its greatest magnitude. The geometrical representation is perfect; the mechanical explanation is not complete. In both respects, as regards what is reduced to law and what is yet incomplete, the Courtown tides must be regarded as the most remarkable that have ever been examined.

Section XVIII.—*Examination into the question of tertio-diurnal tide.*

The observations at Courtown, as has been mentioned, and as appears from the Table in pages 118 and 119, were continued without interruption, day and night, for

a considerable time. The observations at Dunmore East, as appears from the beginning of the Table in page 88, were also continued without interruption for several days. These circumstances appeared to me to offer a convenient opportunity of examining whether the tide occurring three times in the lunar day, which is pointed out by theory, is sensible in the seas around Ireland.

The calculations for this purpose were made in the same manner as the other calculations described in Section X. The means for the twelfth parts of semidiurnal tide, or for the twenty-fourth parts of diurnal tide, having been already found for the operations in Section X., the means of the first and second, of the third and fourth, of the fifth and sixth, &c. were taken; and these were evidently the same as the means for the twelfth parts of diurnal tide. They were then treated by the use of the printed skeleton form shown in Section X., in the same manner as the means for the twelfth parts of semidiurnal tide. Thus the diurnal tide and tertio-diurnal tide were obtained; and a consideration of the principles on which that process is founded will show that the result is in no way affected by the semidiurnal or quarto-diurnal tide. The constants additive to the phase, at Courtown, were corrected where necessary to adapt them to the supposition that the phases are measured from the mean water preceding High Water, First Division, at Ardglass; those at Dunmore East are referred to the same state of tide at Dunmore East. The angle p in the following Table increases by 360° in a tidal day. The day which is set down is that whose astronomical commencement occurs in the tidal day (the limits of the tidal day will be seen in Section X.).

Courtown.			Dunmore East.		
Day.	Diurnal tide.	Tertio-diurnal tide.	Day.	Diurnal tide.	Tertio-diurnal tide.
	ft.	ft.		ft.	ft.
June 29.	$0\cdot57 \cdot \sin (p + 250)$	$0\cdot08 \cdot \sin (3p + 188)$	June 24.	$0\cdot33 \cdot \sin (p + 208)$	$0\cdot16 \cdot \sin (3p + 253)$
30.	$0\cdot31 \cdot \sin (p + 253)$	$0\cdot04 \cdot \sin (3p + 86)$	25.	$0\cdot35 \cdot \sin (p + 207)$	$0\cdot08 \cdot \sin (3p + 304)$
July 1.	$0\cdot43 \cdot \sin (p + 264)$	$0\cdot02 \cdot \sin (3p + 177)$	26.	$0\cdot20 \cdot \sin (p + 192)$	$0\cdot11 \cdot \sin (3p + 220)$
17.	$0\cdot20 \cdot \sin (p + 350)$	$0\cdot14 \cdot \sin (3p + 207)$	27.	$0\cdot27 \cdot \sin (p + 170)$	$0\cdot10 \cdot \sin (3p + 210)$
18.	$0\cdot05 \cdot \sin (p + 201)$	$0\cdot06 \cdot \sin (3p + 236)$	28.	$0\cdot33 \cdot \sin (p + 161)$	$0\cdot09 \cdot \sin (3p + 182)$
19.	$0\cdot24 \cdot \sin (p + 265)$	$0\cdot05 \cdot \sin (3p + 279)$	29.	$0\cdot32 \cdot \sin (p + 150)$	0·00
20.	$0\cdot36 \cdot \sin (p + 261)$	$0\cdot23 \cdot \sin (3p + 187)$	July 1.	$0\cdot19 \cdot \sin (p + 187)$	$0\cdot07 \cdot \sin (3p + 349)$
26.	$0\cdot38 \cdot \sin (p + 256)$	$0\cdot06 \cdot \sin (3p + 179)$	2.	$0\cdot17 \cdot \sin (p + 178)$	$0\cdot03 \cdot \sin (3p + 99)$
27.	$0\cdot43 \cdot \sin (p + 253)$	$0\cdot05 \cdot \sin (3p + 132)$			
28.	$0\cdot49 \cdot \sin (p + 240)$	$0\cdot06 \cdot \sin (3p + 158)$			
29.	$0\cdot29 \cdot \sin (p + 248)$	$0\cdot08 \cdot \sin (3p + 84)$			
30.	$0\cdot25 \cdot \sin (p + 276)$	$0\cdot08 \cdot \sin (3p + 87)$			
31.	$0\cdot15 \cdot \sin (p + 287)$	$0\cdot10 \cdot \sin (3p + 98)$			
Aug. 2.	$0\cdot16 \cdot \sin (p + 10)$	$0\cdot01 \cdot \sin (3p + 306)$			
3.	$0\cdot10 \cdot \sin (p + 105)$	$0\cdot08 \cdot \sin (3p + 315)$			
4.	$0\cdot38 \cdot \sin (p + 106)$	$0\cdot02 \cdot \sin (3p + 3)$			
5.	$0\cdot53 \cdot \sin (p + 87)$	$0\cdot11 \cdot \sin (3p + 328)$			
12.	$0\cdot34 \cdot \sin (p + 57)$	$0\cdot12 \cdot \sin (3p + 346)$			
13.	$0\cdot11 \cdot \sin (p + 357)$	$0\cdot07 \cdot \sin (3p + 238)$			
14.	$0\cdot09 \cdot \sin (p + 46)$	$0\cdot11 \cdot \sin (3p + 211)$			
15.	$0\cdot05 \cdot \sin (p + 286)$	$0\cdot07 \cdot \sin (3p + 264)$			
16.	$0\cdot26 \cdot \sin (p + 286)$	$0\cdot04 \cdot \sin (3p + 70)$			

The diurnal tide is here shown pretty well, and the times of its changes of sign agree well with those found in Section III. But the numbers for tertio-diurnal tide appear to me perfectly lawless. I think that they must be regarded as principally the effects of accident.

On the whole, I am inclined to believe that, as far as evidence goes, the tertio-diurnal tide is not sensible on the coast of Ireland.